MATHEMATICS 9

Module 5: Ratio and Proportion

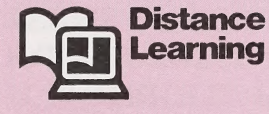
Distance Learning

Alberta EDUCATION

Mathematics 9

Module 5

RATIO AND PROPORTION



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This document is intended for	
Students	✓
Teachers (Mathematics 9)	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 9
Student Module
Module 5
Ratio and Proportion
Alberta Distance Learning Centre
ISBN No. 0-7741-0915-7

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Welcome to Module 5!

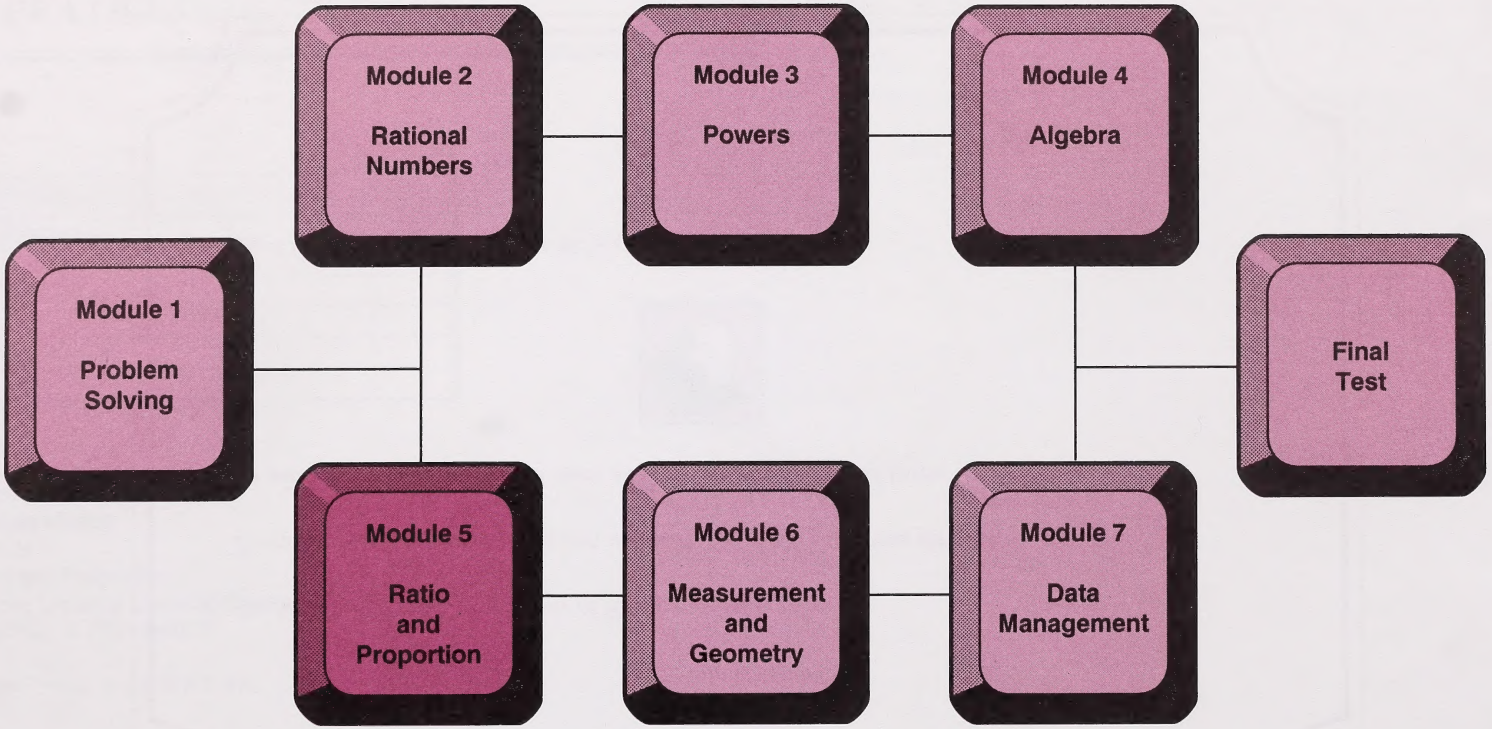
We hope you'll enjoy your study of ***Ratio and Proportion.***

To make your learning a bit easier, a teacher will help guide you through the material.

So whenever you see this icon, turn on your audiocassette and listen.

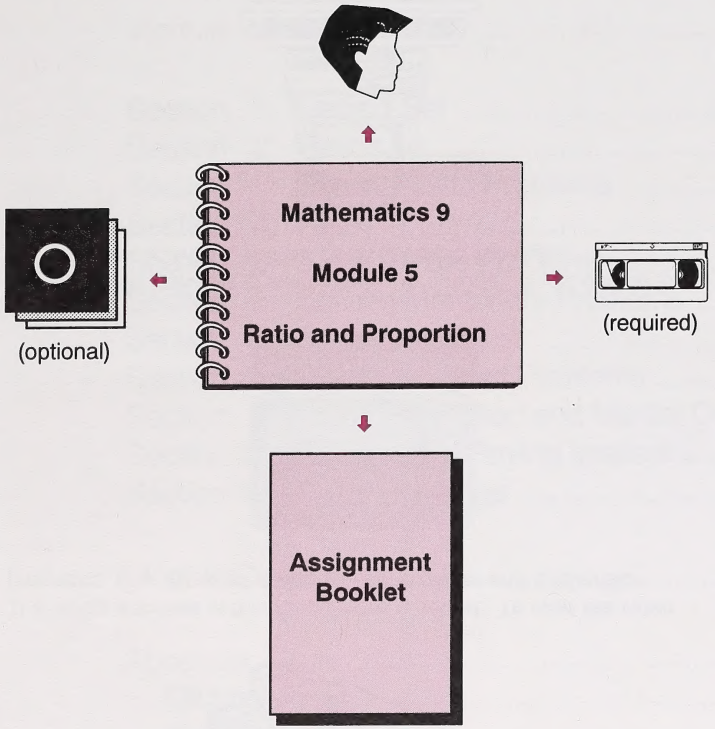
Turn the audiocassette on now to begin.

Course Overview



Mathematics 9 has seven modules and a final supervised test.

Module 5 Components



This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module: the companion audiocassette, the videocassettes, the computer software, and the Assignment Booklet.

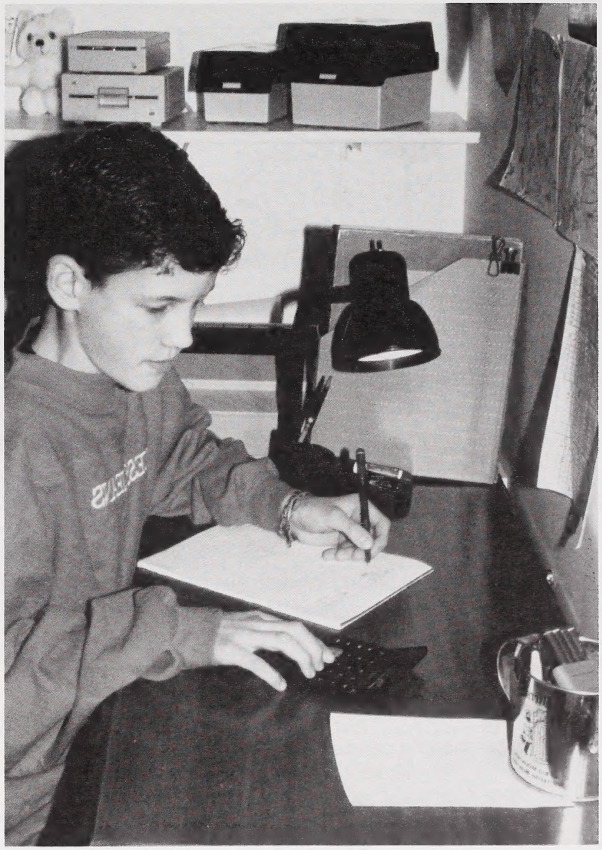
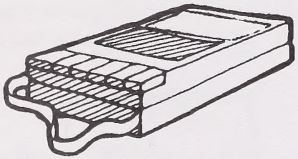


Photo Search Ltd.

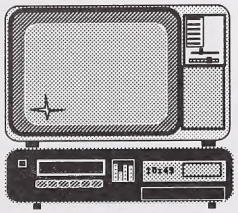
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Optional Equipment

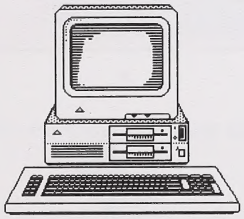
The companion audiocassette for this module is optional. If you decide to listen to it, you will need an audiocassette player.



The video activities in this module are required. To view the video programs, you will need a videocassette player and a television.



The computer activities in this module are optional. If you decide to do the computer activities, you will need an Apple computer.



Required Equipment

You will need a geometry set and a calculator for this module. The calculator should have an $a\frac{b}{c}$ key so that you can perform operations on fractions.

Evaluation

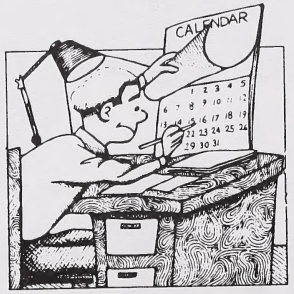
Your mark on this module will be determined by your work in the Assignment Booklet.

Your responses to the questions in this Student Module Booklet are not to be submitted for a grade. However, it is important that you work through the activities carefully before attempting the questions in the Assignment Booklet. This will help you achieve a greater degree of success in your studies.

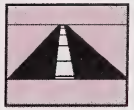
Discuss how the module will be evaluated with your learning facilitator.

Time Management

Decide how long you will need to complete the module. Your learning facilitator will help you plan a schedule.



CONTENTS AT A GLANCE



What Lies Ahead

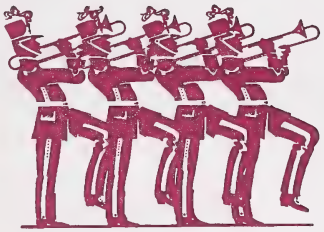
In the module introduction you will preview Module 5.

Working Together

This module will review and extend your knowledge of ratio and proportion. Ratios, rates, and percents are common in everyday life.

Examples of Ratios

- The band class has a ratio of 16 boys to 12 girls.



- For every 12 steps that Jackie takes, Lucy takes 9 steps.
- To make orange juice, Laine uses 3 cans of water for 1 can of orange concentrate.

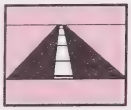
Examples of Rates

- Orest takes 1 spoonful of medicine every 6 h.
- Henrik drove at a rate of speed of 95 km/h on his recent trip to Calgary.

- The sale price is 12 cans for $4.99.

Examples of Percents

- On Saturday there is a 50% discount on all men's shoes.
- Tia scored 83% on her final math exam.
- The real estate agent makes 7% commission on the sale of a house.



What Lies Ahead

This section will pretest the skills taught in this module.

Working Together

The Pretest in this section will help you and your learning facilitator determine your strengths and weaknesses. This will aid you in developing an individualized study plan.

Pretest

1.

a. Write the ratio of the number of nickels to the number of pennies. Use the colon form.

b. Write the ratio of the value of the nickels in cents to the value of the pennies in cents. Use the fraction form.

c. Write the ratio of the value of the nickels in cents to the total value in cents. Use the decimal number form.

2. Two fast food restaurants sell hamburgers. Each of Restaurant A's hamburgers has 29 g of protein and 32 g of fat. Each of Restaurant B's hamburgers has 26 g of protein and 31 g of fat. Which restaurant's hamburgers have the greater ratio of protein to fat?

3. The ratio of Gillian's age to her grandfather's age is 2 to 7. If Gillian is 22 years old, how old is her grandfather?

4. A cranapple drink is made with cranberry juice and apple juice. The ratio of cranberry juice to apple juice is 3 : 2. If 700 mL of cranapple drink is made, how many millilitres of apple juice is used?

5. **a.** Amos can type 99 words in 4 min. Express this as a rate. Use the colon form.

b. Erica sold 49 tires in 7 h. Express this as a rate. Use the fraction form.

6. Francena bought 12 oranges for $3.99. Zoe bought 8 oranges for $2.49. Who received the better price?

7. A car can travel 416 km on 40 L of fuel. How far can it travel on 15 L of fuel?

8. Krish Naidu is travelling from Regina to Prince Albert, a total of 384 km. He starts with a full tank of 85 L. He travels 248 km to Saskatoon and uses 60 L of gasoline. Can he complete the trip to Prince Albert without refuelling?

9. A 1-km long train is travelling at a steady speed of 30 km/h. The front of the train enters a 2-km long tunnel at 1:00 p.m. At what time does the rear of the train emerge from the tunnel?

10. In a package of 100 gummed stars there are 28 blue, 13 gold, 23 green, and 15 silver stars. The rest are red.

a. Express the number of stars of each colour as a percent of the total number of stars.

b. Express the number of gold stars as a percent of the number of green stars.

c. Express the number of blue stars as a percent of the number of silver stars.

11. Express each percent as a decimal number and a fraction in lowest terms.

a. 45% **b.** 9% **c.** 12.5%

d. 160% **e.** $137\frac{1}{2}\%$

12. Express each decimal number as a percent.

a. 0.35 **b.** 0.7 **c.** 1.62 **d.** 4.2

13. Express each fraction as a percent.

a. $\frac{23}{100}$ **b.** $\frac{3}{10}$ **c.** $2\frac{3}{20}$ **d.** $1\frac{7}{25}$

14. A skier won 30% of the races she entered last season. If she entered 80 races, how many did she win?

15. A swimmer won 30 races last season. This was 20% of the races that he entered. How many races did he enter?

16. A baseball team won 30 of the 80 games they played last season. What percent is this?

17. A store is holding a 25% off sale. What is the sale price of a sweater that regularly sells for $45.98?

18. In Alberta a goods and services tax of 7% is added to the price of clothing. What is the final price of a pair of jeans that sells for $39.99?

19. Estimate the answer for each of the following.

a. 40% of 102 **b.** 22% of 10 **c.** $66\frac{2}{3}\%$ of 305

d. 125% of 398 **e.** 104.5% of 85

20. Mentally calculate the answer for each of the following.

a. What is 2% of 32? **b.** What is 30% of 80?

c. What is 75% of 40? **d.** What is $66\frac{2}{3}\%$ of 66?

21. Calculate the interest in each of the following situations.

a. $1250 is invested at 8%/a for 2 a.
b. $800 is borrowed at 12%/a for 6 mo.
c. $3600 is invested at $1\frac{3}{4}\%$/mo for 2 a.

22. The scale of the floor plan of a shopping centre is 1 cm to 4 m. The length of one store on the scale drawing is 4.8 cm. What is the actual length of the store?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- interpreting ratios
- reading and writing ratios
- expressing ratios in lowest terms
- comparing ratios

Working Together

Ratios are frequently used in the everyday world. You have probably encountered ratios like these.

- the win-loss ratio
- the pupil-teacher ratio
- the gear ratio of a bicycle

A **ratio** is a comparison of numbers or quantities measured in the same units.

There are many different ratios that can be used to describe a situation.

Example 1

An aquarium contains 3 orange fish and 5 black fish. Write statements comparing the number of fish in the aquarium.

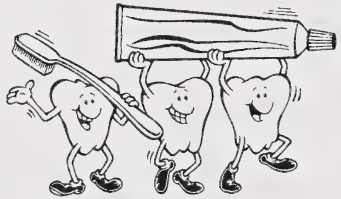
Solution

- The ratio of the number of orange fish to the number of black fish is 3 to 5.
- The ratio of the number of black fish to the number of orange fish is 5 to 3.
- The ratio of the number of orange fish to the total number of fish is 3 to 8.
- The ratio of the number of black fish to the total number of fish is 5 to 8.

Note: When you use ratios, be sure to write a statement that indicates what is being compared. Also make sure that the order of the numbers in the ratio matches the written order of the items being compared.

Example 2

A large tube of toothpaste contains 200 mL. A small tube contains 100 mL. Write statements comparing these quantities.



Solution

- The ratio of the amount of toothpaste in the small tube to the amount of toothpaste in the large tube is 100 to 200.
- The ratio of the amount of toothpaste in the large tube to the amount of toothpaste in the small tube is 200 to 100.
- The ratio of the amount of toothpaste in the small tube to the total amount of toothpaste is 100 to 300.
- The ratio of the amount of toothpaste in the large tube to the total amount of toothpaste is 200 to 300.

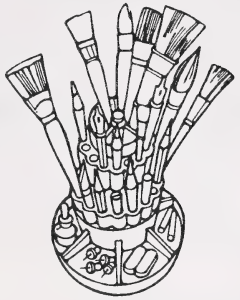
Note: When you are comparing quantities with the same units, the units are not mentioned.

Writing Ratios with Colons

A colon can also be used to express a ratio.

Example

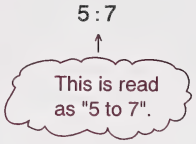
There are 5 pencils and 7 brushes in a container. Write the ratio of the number of pencils to the number of brushes. Use the colon form.



Solution

The ratio of the number of pencils to the number of brushes is 5 to 7.

When written in colon form, the ratio looks like this.



Note: 5 is the **first term** of the ratio and 7 is the **second term** of the ratio.

5 : 7 is a **two-term ratio**.

Writing Ratios in Fraction Form

A ratio can also be expressed in fraction form.

Example

There are 5 soccer balls and 1 football. Write the ratio of the number of footballs to the total number of balls. Use the fraction form.

Solution

The ratio of the number of footballs to the total number of balls is 1 to 6.

The ratio looks like this when written in fraction form.

$\frac{1}{6}$ ← This is read as "1 to 6".

Note: 1 is the first term of the ratio and 6 is the second term of the ratio.

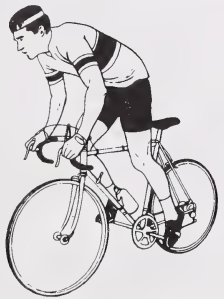
$\frac{1}{6}$ is a two-term ratio.

Writing Ratios in Decimal Number Form

Occasionally, ratios are written in their decimal number form.

Example

The gear ratio of a bicycle compares the number of teeth on the front gear to the number of teeth on the back gear. What is the gear ratio of a ten-speed bicycle in which the front gear has 52 teeth and the back gear has 28 teeth? Use the decimal number form.



Solution

To get the decimal form, first write the ratio as a fraction.

$\frac{52}{28}$

Then use division to change the fraction to a decimal number.

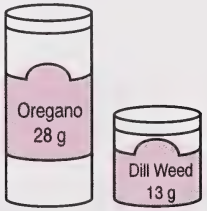
$\frac{52}{28} \doteq 1.86$ ← This is read as "1 point eight six".

The gear ratio is about 1.86.

Note: The gear ratio of 1.86 means that the ratio of the number of teeth on the front gear to the number of teeth on the back gear is 1.86 to 1. The second term, 1, is understood.

Practice Activity 1

1. Write statements comparing the capacities of these spice bottles. Use the ratios that follow.



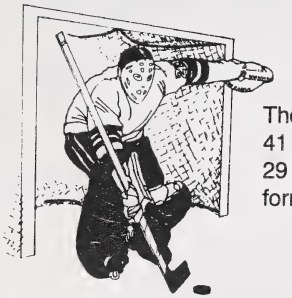
a. 28 to 13 **b.** 13 to 41 **c.** 28 to 41

2. There are 7 balls in a group of 10 items of sports equipment. What is the ratio of the number of balls to the total number of equipment items? Use the fraction form.

3. There are 5 roses and 8 daisies in a bouquet of flowers. Use the colon form to write the following ratios.

   a. the number of roses to the number of daisies
   b. the number of roses to the total number of flowers

4. There are 128 students and 5 teachers in a school. Write the student-teacher ratio. Use the decimal number form rounded to the nearest tenth.

5. 

   The Hawks hockey team won 41 games, lost 10 games, and tied 29 games. Use the decimal number form to write the win-loss ratio.

Turn to the Appendix to check your answers.

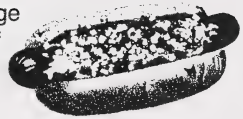
Working Together

Writing Ratios in Lowest Terms

Ratios are usually written in **lowest terms**. The terms of the ratios should be the lowest whole numbers possible.

Example 1

Yvonne has a package of 8 buns and a package of 12 wieners. Write the ratio of the number of buns to the number of wieners. Express the ratio in lowest terms.



Solution

Write the ratio of the number of buns to the number of wieners.

$\frac{8}{12}$ $\frac{\text{Number of buns}}{\text{Number of wieners}}$

To express the ratio in lowest terms, divide each term by 4.

$$\frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$$

The ratio of the number of buns to the number of wieners is 2 to 3.

Note: 8 to 12 and 2 to 3 are **equivalent ratios** or **proportional ratios**.

Example 2

In a punch recipe there are 2.7 L of pineapple juice and 1.3 L of orange juice. What is the ratio of orange juice to pineapple juice? Express the ratio in lowest terms.

Solution

Write the ratio of orange juice to pineapple juice.

$1.3 : 2.7$

To express the ratio in lowest terms, multiply each term by 10. This is necessary because the lowest terms must be whole numbers.

$10 \times 1.3 : 10 \times 2.7 = 13 : 27$

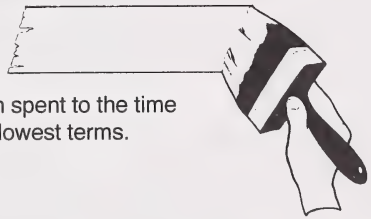
The ratio of orange juice to pineapple juice is 13 to 27.

Example 3

Jason painted the fence for $1\frac{3}{4}$ h.

Ruth painted the fence for $1\frac{1}{2}$ h.

What is the ratio of the time Jason spent to the time Ruth spent? Express the ratio in lowest terms.



Solution

Write the ratio of the time Jason spent to the time Ruth spent.

$$1\frac{3}{4} : 1\frac{1}{2}$$

To express the ratio in lowest terms, first express the terms as improper fractions.

$$\frac{7}{4} : \frac{3}{2}$$

Next express the terms as equivalent fractions with common denominators.

$$\frac{7}{4} : \frac{3}{2} = \frac{7}{4} : \frac{6}{4}$$

To express the ratio in lowest terms, multiply by 4.

$$4 \times \frac{7}{4} : 4 \times \frac{6}{4} = 7 : 6$$

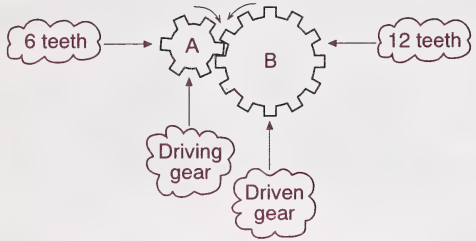
The ratio of the time Jason spent to the time Ruth spent is 7 to 6.

Practice Activity 2

1. A neighbourhood has 28 cats and 16 dogs. What is the ratio of the number of cats to the number of dogs in the neighbourhood? Express the ratio in lowest terms.

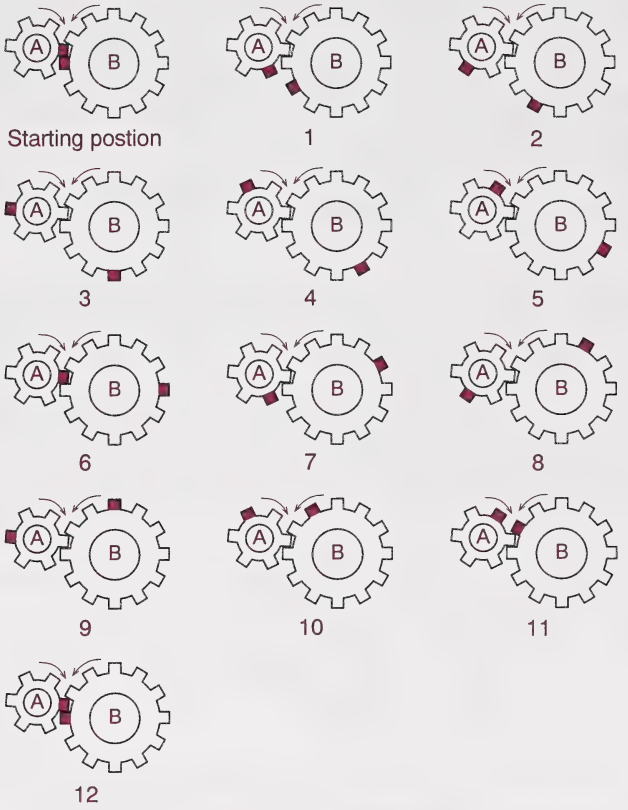
2. The weather report said that in Alberta the temperature was 4°C and in British Columbia the temperature was 10°C. Write the ratio of the temperature in Alberta to the temperature in British Columbia. Express the ratio in lowest terms.

3. **a.** Examine this diagram of a gear train. Notice that there are two gears. Gear A is the driving gear. Gear B is the driven gear.



What is the gear ratio? **Hint:** The gear ratio is the ratio of the number of teeth on the driving gear to the number of teeth on the driven gear.

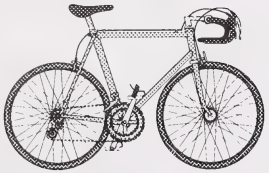
b. Notice how the teeth of the two gears mesh. As Gear A moves, it drives Gear B. Also notice the direction each gear is moving. Gear A moves clockwise and Gear B moves counterclockwise. Examine the position of the coloured teeth on each of the gears in the following diagram. How many revolutions (complete turns) do Gear A and Gear B each make?



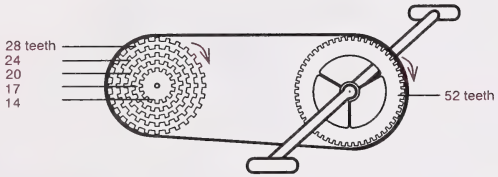
c. What is the turn ratio? **Hint:** The turn ratio is the ratio of the number of turns of the driving gear to the number of turns of the driven gear.

d. What do you notice about the gear ratio and the turn ratio?

4. On a five-speed bicycle, a chain connects the front gear and the back gear.



Below is a diagram of the gears on a 10-speed bicycle.



Notice that there is one gear on the pedal and there are five gears on the back wheel. The gears can be connected in different positions. Also notice that the front and back gears move in the same direction.

a. Complete a chart like this . Express the gear ratios and turn ratios in lowest terms.

Gear	Teeth on Front Gear	Teeth on Back Gear	Gear Ratio	Turn Ratio
1st	52	28		
2nd	52	24		
3rd	52	20		
4th	52	17		
5th	52	14		

b. What do you notice about the gear ratio and turn ratio for each gear?

c. In what gear will the bicycle be easiest to pedal?

d. In which gear will the bicycle go fastest?

5. The body of an adult contains about 4.7 L of blood. A blood donor usually gives 0.47 L of blood. What is the ratio of the quantity of donated blood to the quantity of blood in the body? Express the ratio in lowest terms.

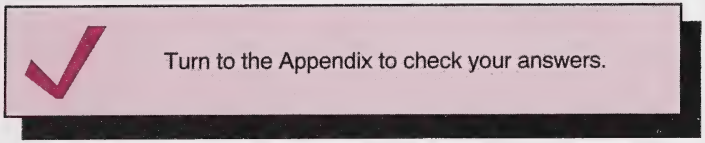
6. In one day an adult inhales 11.4 cm^3 of air. About 0.57 cm^3 of oxygen in this air is absorbed into the bloodstream. What is the ratio of the volume of oxygen absorbed to the volume of air inhaled? Express the ratio in lowest terms.

7. A Minute Beetle is 0.02 cm long. A Goliath Beetle is 14.86 cm long. What is the ratio of the length of the Minute Beetle to the length of the Goliath Beetle? Express the ratio in lowest terms.

8.

Brian slept for $6\frac{1}{2}$ h on Monday night and $7\frac{1}{3}$ h on Tuesday night. What is the ratio of the time he slept on Monday night to the time he slept on Tuesday night? Express the ratio in lowest terms.

9. The mass of precious jewels is measured in carats (c). The largest blue diamond in the world, the Hope Diamond, has a mass of $44\frac{2}{5}$ c. Another famous diamond, the Timkin Stone, has a mass of $28\frac{3}{4}$ c. What is the ratio of the mass of the Hope Diamond to the mass of the Timkin Stone? Express the ratio in lowest terms.

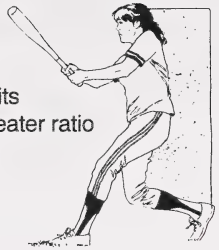


Comparing Ratios

Ratios are often compared.

Example

Talia hits safely 5 out of 8 times at bat. René hits safely 7 out of 10 times at bat. Who has the greater ratio of hits to times at bat?



Method 1

First write the ratios of hits to times at bat.

$$\frac{\text{Number of hits}}{\text{Number of times at bat}}$$

René	Talia
$\frac{7}{10}$	$\frac{5}{8}$

Then use the least common multiple of the second terms to write equivalent ratios.

René: $\frac{7}{10} \xrightarrow{\times 4} \frac{28}{40}$ Talia: $\frac{5}{8} \xrightarrow{\times 5} \frac{25}{40}$

$$\frac{28}{40} > \frac{25}{40}$$

So, $\frac{7}{10} > \frac{5}{8}$.

Therefore, René has the greater ratio of hits to times at bat.

Method 2

First write the ratios of hits to times at bat.

$$\frac{\text{Number of hits}}{\text{Number of times at bat}}$$

René	Talia
$\frac{7}{10}$	$\frac{5}{8}$

Then use the product of the second terms to write equivalent ratios.

René: $\frac{7}{10} \xrightarrow{\times 8} \frac{56}{80}$ Talia: $\frac{5}{8} \xrightarrow{\times 10} \frac{50}{80}$

$\frac{56}{80} > \frac{50}{80}$

So, $\frac{7}{10} > \frac{5}{8}$.

René has the greater ratio of hits to times at bat.

Method 3

First write the ratios of hits to times at bat.

Number of hits / Number of times at bat

René	Talia
$\frac{7}{10}$	$\frac{5}{8}$

Find the cross products. This is a short-cut method of finding the first terms in Method 2.

$7 \times 8 = 56$ $\qquad$ $5 \times 10 = 50$

$\frac{7}{10} \quad \frac{5}{8}$

Then compare the cross products.

$56 > 50$

So, $\frac{7}{10} > \frac{5}{8}$.

René has the greater ratio of hits to times at bat.

Method 4

Change the ratios to their decimal number forms.

René	Talia
$\frac{7}{10} = 0.7$	$\frac{5}{8} = 0.625$

Then compare the decimal numbers.

$0.7 > 0.625$

So, $\frac{7}{10} > \frac{5}{8}$.

René has the greater ratio of hits to times at bat.

Note: A calculator can also be used to find the decimal number forms.

René

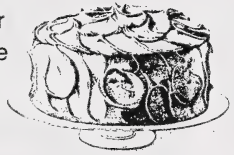
Key Press	Display
7 ÷ 1 0 =	0.7

Talia

Key Press	Display
5 ÷ 8 =	0.625

Example 2

In a cake recipe the ratio of the measure of flour to the measure of sugar is 3 : 1. In another cake recipe the ratio of the measure of sugar to the measure of flour is 3 : 1. Which recipe makes sweeter cakes?



Solution

The recipe with the greater ratio of sugar to flour will be sweeter.

The ratios in the problem are not written in the same order. Rewrite the ratios so that they are comparing the amount of sugar to the amount of flour.

$\frac{\text{Sugar}}{\text{Flour}}$

Recipe 1	Recipe 2
$\frac{1}{3}$	$\frac{3}{1}$

Now compare the ratios.

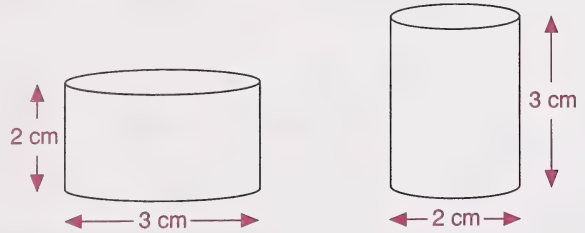
$$\frac{1}{3} < \frac{3}{1}$$

So, the second recipe is sweeter.

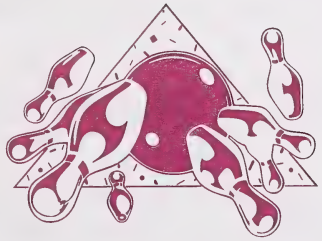
Practice Activity 3

1. Look at the photographs to the right. Compare the ratios of the lengths to the widths. Are they equivalent?

2. Look at these cylinders. Compare the ratios of the heights to the diameters. Are they equivalent?



3. Last year a bowling team won 90 out of 162 games. So far this year they have won 10 out of 12 games. Is the team doing better or worse than it did last year?



4. In four games a basketball player made 7 out of 10 shots, 14 out of 20 shots, 3 out of 5 shots, and 3 out of 4 shots.

a. In which game did the player have the greatest success?

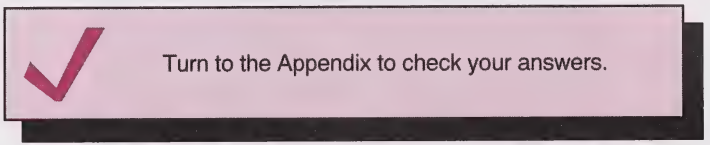
b. In which game did the player have the least success?

c. In which games did the player have the same amount of success?

5.

Victor and Aaron play basketball. Victor sank 6 out of 9 shots. Aaron sank 9 out of 15. Which of these players is the more accurate shooter?

6. Amelia scored 17 out of 25 on the first test and 14 out of 20 on the second test. On which test did she do better?



Working Together

This part of the section is included for enrichment.

So far all of the ratios you have worked with have had two terms. You will now learn about ratios that have more than two terms.

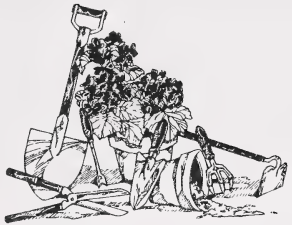
Example

A bag of potting soil contains 5 parts of loam, 2 parts of peat moss, and 1 part of coarse sand. Different **three-term ratios** can describe this situation.

- The ratio of loam to peat moss to coarse sand is 5 to 2 to 1.
- The ratio of loam to coarse sand to peat moss is 5 to 1 to 2.
- The ratio of peat moss to coarse sand to loam is 2 to 1 to 5.
- The ratio of peat moss to loam to coarse sand is 2 to 5 to 1.
- The ratio of coarse sand to peat moss to loam is 1 to 2 to 5.
- The ratio of coarse sand to loam to peat moss is 1 to 5 to 2.

The total number of parts of potting soil is $5 + 2 + 1 = 8$. So, different two-term ratios can be written as well.

- The ratio of loam to potting soil is 5 to 8.
- The ratio of peat moss to potting soil is 2 to 8 or 1 to 4.
- The ratio of coarse sand to potting soil is 1 to 8.

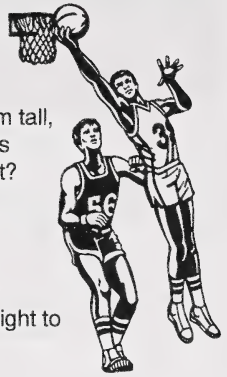


Three-term ratios can be written using words or the colon form.

Two-term ratios can be written using words, the colon form, the fraction form, or the decimal number form.

Three-term ratios can also be written in lowest terms.

Example



Franz's father is 175 cm tall, his mother is 160 cm tall, and he is 200 cm tall. What is the ratio of Franz's height to his father's height to his mother's height? Express the ratio in lowest terms.

Solution

Write the ratio of Franz's height to his father's height to his mother's height.

$200 : 174 : 160$

To find the lowest terms of the ratio, you must divide all the terms of the ratio by their greatest common factor.

The greatest common factor of 200, 175, and 160 is 5. So, divide each term by 5.

$$200 : 175 : 160 = 200 \div 5 : 175 \div 5 : 160 \div 5$$
$$= 40 : 35 : 32$$

The ratio of Franz's height to his father's height to his mother's height is 40 to 35 to 32.

Practice Activity 4

1. Martejka is mixing different colours of paint to create a shade of brown. To make this shade, she mixes 8 parts of red paint, 5 parts of yellow paint, and 1 part of black paint. Express the ratios in lowest terms.

   a. What is the ratio of the quantity of red to yellow to black?
   b. What is the ratio of the quantity of black to red to yellow?
   c. What is the ratio of the quantity of yellow to red to black?
   d. What is the ratio of the quantity of yellow to brown?
   e. What is the ratio of the quantity of black to brown?
   f. What is the ratio of the quantity of red to brown?

2. Albert is 12 years old, Mavis is 13 years old, and Emma is 15 years old.

   a. What is the ratio of Albert's age to Mavis' age to Emma's age?

   b. What is the ratio of Emma's age to Mavis' age to Albert's age?

   c. What is the ratio of Albert's age to the total age of all three children?

   d. What is the ratio of the sum of Mavis' age and Albert's age to the total age of all three children?

   e. What is the ratio of Emma's age to the total age of all three children?

3. To make a cement foundation, 300 kg of gravel are mixed with 450 kg of sand and 100 kg of cement. What is the ratio of the mass of gravel to sand to cement? Express the ratio in lowest terms.

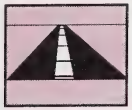
4. During a hockey season a team had 36 wins, 24 losses, and 8 ties. What is the ratio of wins to losses to ties? Express the ratio in lowest terms.

5. At the 1988 Winter Olympics in Calgary, West Germany won 2 gold medals, 4 silver medals, and 2 bronze medals. Switzerland won 5 gold medals, 5 silver medals, and 5 bronze medals.

   a. What is the ratio of the number of gold medals to silver medals to bronze medals won by West Germany? Express the ratio in lowest terms.

   b. What is the ratio of the number of gold medals to silver medals to bronze medals won by Switzerland? Express the ratio in lowest terms.

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will solve ratio problems.

Working Together

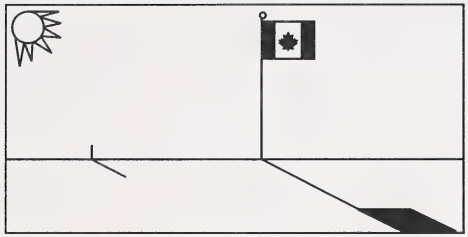
In the previous section you learned about equivalent or proportional ratios.

In this section you will learn about proportions. A **proportion** is an equation that shows the equality of two ratios.

Proportions are very helpful. They allow you to find a missing term of a ratio if you are given one term of the ratio and a proportional ratio.

Example 1

Malachi measured the shadow of a metre stick and the shadow of a flagpole at the same time of day. The shadow of the metre stick was 3 m long. The shadow of the flagpole was 15 m long. How high is the flagpole?



Solution

Write the proportion. Use a variable for the missing term.

$$\frac{\text{Height of object in m}}{\text{Length of its shadow in m}}$$

Metre stick		Flagpole
$\frac{1}{3}$	=	$\frac{h}{15}$

Use a method of your choice to solve for h. The cross-product method is shown.

$$15 = 3h$$

$$\frac{1}{\cancel{3}} \times \frac{\cancel{15}^{5}}{1} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}^{1}h}{1}$$

$$5 = h$$

So, the height of the flagpole is 5 m.

Example 2

As Marion walks with her little sister, Kyra, she takes 4 steps for every 5 steps that Kyra takes. How many steps does Kyra take if Marion takes 50 steps?

Solution

Write the proportion. Use a variable for the missing term.

$$\frac{\text{Marion's steps}}{\text{Kyra's steps}} \qquad \frac{4}{5} = \frac{50}{k}$$

Use a method of your choice to solve for k. The cross-product method is shown.

$$4k = 250$$

$$\frac{1}{\cancel{4}_{1}} \times \cancel{4}^{1} k = \frac{1}{\cancel{4}_{2}} \times \cancel{250}^{125}$$

$$k = \frac{125}{2}$$

$$k = 62.5$$

So, Kyra takes 62.5 steps when Marion takes 50 steps.

Example 3

If the ratio of domestic cars to foreign cars in a city is 9 to 5, how many foreign cars can you expect to find in a parking lot holding 280 cars?

Solution

Since $9 + 5 = 14$, the ratio of the number of foreign cars to the total number of cars is 5 to 14.

Write a proportion.

$$\frac{\text{Foreign cars}}{\text{Total cars}} \qquad \frac{5}{14} = \frac{n}{280}$$

Use a method of your choice to solve for n. The cross-product method is shown.

$$1400 = 14n$$

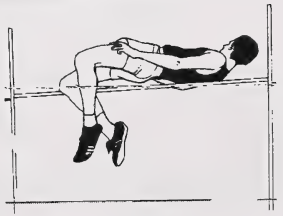
$$\frac{1}{\cancel{14}_{1}} \times \cancel{14}^{1} n = \frac{1}{\cancel{14}_{1}} \times \cancel{1400}^{100}$$

$$100 = n$$

So, you can expect to find 100 foreign cars in the parking lot.

Practice Activity 1

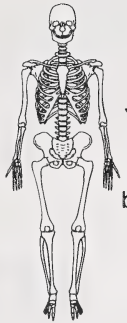
1. The moon pulls objects towards it with a different amount of force than the earth does. It is easier to jump away from the surface of the moon than it is to jump away from the surface of the earth. The ratio of a person's jump on Earth to a person's jump on the moon is 1 to 6.



a. If a high jumper can jump 2.25 m on Earth, how high could he jump on the moon?

b. If a pole jumper can jump 5.5 m on Earth, how high could she jump on the moon?

2.



Your body contains 206 bones. The ratio of the number of bones in your head to the total number of bones in your body is about 1 to 7. About how many bones are there in your head?

3. The amount of gold in jewellery is measured in karats (K). The mark of 24 K on a bracelet means the bracelet is pure gold. The mark of 14 K on a bracelet means the ratio of the mass of gold in a bracelet to the mass of the bracelet is $14:24$. If the total mass of a 14-K bracelet is 52 g, what is the mass of the gold in the bracelet?

4. **a.** When a ten-speed bicycle is in first gear, 7 revolutions of the pedal make 10 revolutions of the wheel. How many pedal revolutions are needed for 120 wheel revolutions?

b. When a ten-speed bicylce is in tenth gear, 1 revolution of the pedal makes 3 revolutions of the wheel. How many pedal revolutions are needed for 120 wheel revolutions?

5. Denzil is a quarterback. His ratio of the number of passes completed to the number of passes attempted is $2:5$. In a game Denzil completed 12 passes. How many passes did he attempt?

6. The ratio of the mass of a hydrogen atom to the mass of an average man is about the same as the ratio of the mass of an average man to the mass of the sun. The mass of a hydrogen atom is about 1.7×10^{-29} kg. The mass of an average man is about 70 kg. What is the approximate mass of the sun?

Note: Question 6 requires a knowledge of operations with numbers expressed in scientific notation. This is taught in Module 3, but is optional content.

Turn to the Appendix to check your answers.

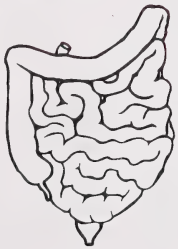
Working Together

In Module 1 you learned that you can simplify a problem by breaking the problem into steps or subproblems.

In the Practice Activitiy that follows, you will need to break the problems into subproblems and tackle one subproblem at a time.

Practice Activity 2

1.



The ratio of the length of your small intestine to the length of your large intestine is about 4 : 1. If your large and small intestines are about 7.6 m long in total, how long is your small intestine?

2. Sterling silver is an alloy of silver and copper in the ratio of 37 to 3. How many grams of silver are there in a sterling silver goblet with a total mass of 500 g?

3. Julie has two bags with the same number of marbles in each bag. In Bag A the ratio of black marbles to red marbles is 3 to 4. In Bag B the ratio of black marbles to red marbles is 2 to 5. If Julie has only red marbles and black marbles, and if there are 70 marbles altogether in the two bags, find the following.

a. the number of black marbles in Bag A
b. the number of red marbles in Bag A
c. the number of black marbles in Bag B
d. the number of red marbles in Bag B

4. Originally, the ratio of the number of girls to the number of boys in Ingrid's chess club was 5 to 7. When 4 new girls joined the club, the ratio of the number of girls to the number of boys became 1 to 1. What was the final number of boys and girls in the chess club?

5. Sylvia and Jane are partners in a small business. Originally Sylvia invested $70 and Jane invested $50. They agreed to share the profits in the ratio of 7 to 5. If the business makes $1500 profit this month, what is Sylvia's share?

Turn to the Appendix to check your answers.

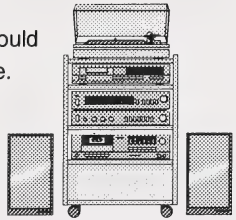
Working Together

This part of the section is included for enrichment.

You can use proportions to find a missing term in a three-term ratio.

Example 1

If stereo speakers are to have good acoustics (sound), the ratio of depth to width to height should be 1 : 2 : 3. Rial made his speakers 60 cm wide. What is the height and depth of the speakers?



Solution

Three ways to solve this problem are shown.

Method 1

Write the three-term ratios as a proportion. Use boxes for the unknown terms in the second ratio.

Depth : Width : Height

1 : 2 : 3

= ☐ : 60 : ○

By comparing, you can see that 2 was multiplied by 30 to get 60. Therefore, the other known terms can also be multiplied by 30 to solve for the unknown terms.

1 : 2 : 3

× 30 × 30 × 30

= 30 : 60 : 90

So, the depth is 30 cm and the height is 90 cm.

Method 2

Write the three-term ratios as a proportion. Use variables for the unknown terms.

Depth : Width : Height

1 : 2 : 3

= d : 60 : h

To find the depth, write a proportion using the first and second terms of each ratio.

$$\frac{1}{d} = \frac{2}{60}$$

Use a method of your choice to solve for d. The cross-product method is shown.

$$60 = 2d$$

$$\frac{1}{\cancel{2}_{1}} \times \cancel{60}^{30} = \frac{1}{\cancel{2}_{1}} \times \cancel{2}^{1} d$$

$$30 = d$$

So, the depth is 30 cm.

To find the height, write a proportion using the second and third terms of each ratio.

$$\frac{2}{60} = \frac{3}{h}$$

Use a method of your choice to solve for h. The cross-product method is shown.

$$2h = 180$$

$$\frac{1}{\cancel{2}_{1}} \times \cancel{2}^{1} h = \frac{1}{\cancel{2}_{1}} \times \cancel{180}^{90}$$

$$h = 90$$

So, the height is 90 cm.

Practice Activity 3

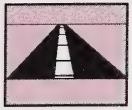
1. The ratio of the number of red marbles to blue marbles to white marbles in a bag is $5:12:8$. If there are 100 red marbles in the bag, how many blue and white marbles are there?

2. Three partners invest in a company in the ratio of $5:4:1$. If they divide their profits of \$150 000 in the same ratio as they invested, how much does each partner receive?

3. **a.** A 1-cent coin minted before 1860 had a mass of 4.5 g and contained copper, tin, and zinc in the ratio of $95:4:1$. What mass of copper did the coin contain?

   b. After 1942, each 1-cent coin had a mass of 3.24 g and contained copper, tin, and zinc in the ratio $98:0.5:1.5$. What mass of copper did the coin contain?

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will learn these skills.

- interpreting rates
- reading and writing rates
- simplifying rates
- comparing rates

Working Together

Rates are frequently used in the everyday world. A **rate** compares quantities measured in different units.

Example 1

The bus from Edmonton to Calgary travels a distance of 300 km in 3 h.

The rate of speed can be expressed in different ways.

- The rate of speed is $300\ \text{km} : 3\ \text{h}$.
- The rate of speed is 300km/3 h.
- The rate of speed is $\frac{300\ \text{km}}{3\ \text{h}}$.

These expressions are all read as "300 km per 3 h".

Note: Speed rates express the ratio of distance to time. Because the units are different, they must be included in the ratio.

Example 2

The cost of 0.865 kg of chicken is $5.18.

The price rate can also be expressed several ways.

- The price rate is \$5.18 for 0.865 kg.
- The price rate is $\$5.18 : 0.865\ \text{kg}$.
- The price rate is \$5.18/0.865 kg.
- The price rate is $\frac{\$5.18}{0.865\ \text{kg}}$.

These expressions are all read as "\$5.18 per 0.865 kg".

Note: Price rates express the ratio of price to mass. Because the units are different, they must be included in the ratio.

Practice Activity 1

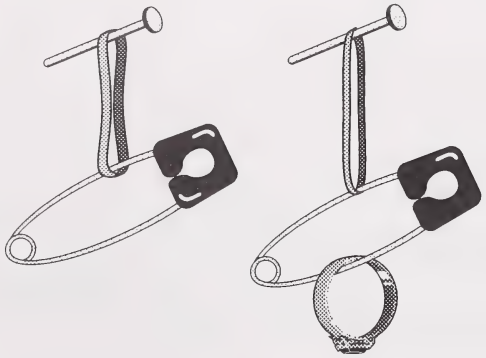
1. Write rates to describe the following situations. Use the colon form.

a. A child buys 2 apples for \$0.75.

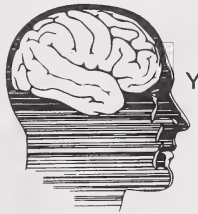
b. A typist types 721 words in 8 min.

c. A metal expands 12.7 cm as the temperature rises 18°C.

d. A rubber band stretches 0.5 cm when a 15-g mass is hung on it.



e.



Your brain sorts out 10^5 messages every second.

2. Write rates to describe the statements in Question 1. Use the fraction form.

Turn to the Appendix to check your answers.

Working Together

Simplifying Rates

Usually rates are simplified. One way to simplify a rate is to write the rate with a second term of 1.

Example

It rained 81.6 mm in 5 h. What was the hourly rate of rainfall?

Solution

Write the rate of rainfall in 5 h.

81.6 mm/5h

The find the hourly rate by dividing each term by 5.

$$\frac{81.6}{5} \overset{\div 5}{=} \frac{16.32}{1}$$

The rate of rainfall is 16.32 mm/h.

Comparing Rates

Writing rates with a second term of 1 makes it easier to compare the rates.

Example

Jeanne earns $56 in 8 h. Malene earns $48 in 6 h. Who earns more money per hour?

Solution

Write each rate of pay.

Jeanne	Malene
$56/8 h	$48/6 h

Write each hourly rate of pay.

Jeanne

$$\frac{56}{8} \xrightarrow{\div 8} \frac{7}{1}$$

Jeanne earns $7/h.

Malene

$$\frac{48}{6} \xrightarrow{\div 6} \frac{8}{1}$$

Malene earns $8/h.

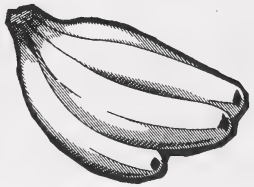
Compare the rates.

$8/h > $7/h

So, Malene earns more money per hour.

Practice Activity 2

1. Write the following statements as rates in simplest form.
   - **a.** Evan's heart beats 300 times in 4 min.
   - **b.** Bob can type 420 words in 4 min.
   - **c.** A heavy rubber band stretches 4 cm when supporting a mass of 6 g.
2. Density is the mass per cubic centimetre of a substance. For example, 1 cm^3 of water has a mass of 1 g, so the density is 1 g/cm^3.
   - **a.** 55 cm^3 of mercury has a mass of 748 g. What is its density?
   - **b.** 50 cm^3 of cast iron has a mass of 350 g. What is its density?
   - **c.** 60 cm^3 of oak has a mass of 40 g. What is its density?
3. Which is the better buy, 5 kg of bananas for $7.35 or 3 kg of bananas for $4.45?



4. Dino travelled 830 km in 9 h. Frank travelled 640 km in 7 h. Who travelled at a faster rate of speed?

5. An athlete runs 100 m in 9.9 s. Another athlete runs 200 m in 19.8 s. Which athlete runs faster?

Turn to the Appendix to check your answers.

Working Together

Not all rates are written with a second term of 1.

Example

Gaston's car used 68 L in a 500-km trip. What is his rate of fuel consumption? **Hint:** Fuel consumption is measured in litres per hundred kilometres.

Solution

The rate is 68 L/500 km. Calculate the litres per 100 km.

$$\frac{68}{500} \overset{\div 5}{=} \frac{13.6}{100}$$

Gaston's gas consumption is 13.6 L/100 km.

Example 2

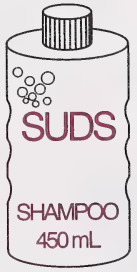
Shampoo costs $3.79 for 450 mL. What is the cost for 100 mL?

Solution

The cost is $3.79/450 mL.

Calculate the cost for 100 mL.

$$\frac{3.79}{450} \overset{\div 4.5}{=} \frac{0.84}{100}$$



The shampoo costs about $0.84/100 mL.

Example 3

Salami costs $4.45 for 500 g. What is the cost for 100 g?

Solution

The cost is $4.45/500 g.

Calculate the cost for 100 g.

$$\frac{4.45}{500} \overset{\div 5}{=} \frac{0.89}{100}$$

The salami costs $0.89/100 g.

Example 4

In 1985 there were 61 980 divorces in Canada. If the population in Canada in 1985 was about 25 360 000, what was the divorce rate?

Solution

The divorce rate in Canada in 1985 was 61 980 divorces per 25 360 000 population.

Calculate the number of divorces per 1000 population.

$$\frac{61\,980}{25\,360\,000} \overset{\div 25\,360}{=} \frac{2.44}{1000}$$

In Canada in 1985 there were 2.44 divorces per 1000 population.

Note: The birth rates, death rates, and marriage rates in Canada are also measured in frequencies per 1000 population.

Practice Activity 3

1.

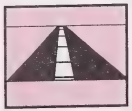
Miguel's car travels 800 km on 72 L of gas. Rico's car travels 350 km on 33 L of gas. Whose car has the better rate of gasoline consumption? Express the consumption rates as L/100 km.

2. A 750-mL bottle of pop sells for $1.29. A 2-L bottle of pop sells for $2.99. Which is the better buy? Express the costs as $/100 mL. (**Hint:** 1 L = 1000 mL)

3. Ruth bought 500 g of smoked turkey at the deli for $6. Yvonne bought 750 g of smoked turkey at the grocery store for $9.30. Who received the better buy? Express the costs as $/100 g.

4. There were 185 597 marriages in Canada in 1985. If the population in Canada was about 25 360 000, what was the marriage rate?

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will solve rate problems.

Working Together

In Section 3 you used proportions to solve problems involving ratios. In this section you will use proportions to solve problems with rate.

Example

Jessica is watching a storm. She sees lightning and 8 s later she hears thunder. How far away is the lightning? (**Hint:** Sound travels at a rate of 344 m/s.)

Solution

Write the rate of speed at which sound travels.

344 m/s

Use this rate to write a proportion. Use a variable for the missing term.

$$\frac{\text{Distance (m)}}{\text{Time (s)}} \qquad \frac{344}{1} = \frac{d}{8}$$

Use a method of your choice to solve for d. The cross-product method is shown.

$$\frac{344}{1} = \frac{d}{8}$$
$$344 \times 8 = d$$
$$2752 = d$$

The lightning is 2752 m away.

Practice Activity 1

1. Arthur is training for a 10-km race. He jogs 6 km daily. It takes him an average of 30 min to jog this distance.

- **a.** What is Arthur's rate of speed while training?
- **b.** At this rate, how long will it take him to finish the 10-km race?
- **c.** At this rate, how far can he jog in 45 min?

2. Complete a table like this.

Kilograms of Meat	2	4	
Cost in Dollars	8.60		43.00

3. If 8 pairs of socks sell for $10.99, what do 10 pairs cost?

4. The human heart beats an average of 72 times per minute. At this rate, how many heartbeats are there in 30 min?

5. The table below lists the cruising speeds of three planes.

Type of Plane	Cruising Speed (km/h)
DC-9	826
747	893
L-1011	882

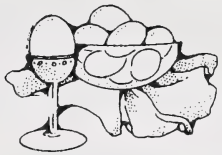
For each plane, determine how far it can travel in 3 h.

6. René, who works a 40-h week, is paid $450 per week. At the same hourly wage, how much will she receive if she works a 36-h week?

7. A bicyclist travels 28.4 km in 2.25 h. At that rate, how long will it take to complete 50 km?

8. At a rate of 2.5 m/s, how far can Arlene swim in 50 s?

9. Jackson's fuel consumption is 17 L per 100 km. At that rate, how much gasoline would be needed for a 2500-km trip?

10. 

If it takes 3 min to boil one egg, how long does it take to boil 4 eggs?

Turn to the Appendix to check your answers.

Working Together

In Module 1 you learned that you can simplify a problem by breaking the problem into steps or subproblems.

In the Practice Activity that follows, you will need to break the problem into subproblems and tackle one subproblem at a time.

Practice Activity 2

1. Your eye blinks about 25 times each minute. About how many times does your eye blink in an hour?

2. Your heart beats about 80 times in a minute. About how many times does your heart beat in a day?

3. The very slow giant tortoise of Mauritius travels at a speed of about 5 m/min. How long will it take to travel 5 km?

4. Gino can run at 5 m/s. An athlete can run at 7 m/s. If Gino has a head start of 5 s, would he win a 100-m race?

5. Raja's economy car consumes fuel at the rate of 6 L/100 km. Helmut's van consumes fuel at the rate of 15 L/100 km. How many litres of fuel must Helmut's van consume to travel as far as Raja's car travels on 60 L?

6. A swimmer's best record for the 100-m freestyle is 51.2 s. Her best record for the 200-m freestyle is 1 min 52.8 s. If she could swim the 200-m race at the same rate as the 100-m race, how much time would she cut from her record?

7. Larissa is paid $8.60/h for a 40-h week, and she earns $14.75/h overtime. How much does she earn for a 50-h week?

Turn to the Appendix to check your answers.

Working Together

This part of the section is included for enrichment.

Some rate problems involve more than one person, machine, or animal doing a job.

Example 1

Jim can knit 1 set of golf-club covers in 3 d. Joe can knit 1 set in 6 d. If they work together, how long will it take them to knit a set of golf-club covers?

Solution

The boys are working at different ratios.

Jim can knit 1 set in 3 d.

Jim can knit $\frac{1}{3}$ of a set in 1 d.

Joe can knit 1 set in 6 d.

Joe can knit $\frac{1}{6}$ of a set in 1 d.

Calculate the daily rate if they work together.

$$\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}$$
$$=\frac{3}{6}$$
$$=\frac{1}{2}$$

Jim and Joe working together can knit $\frac{1}{2}$ of a set in 1 d.

Write a proportion to find the number of days needed to knit one set if they work together at this rate.

$$\frac{\text{Number of sets}}{\text{Number of days}} \qquad \frac{\frac{1}{2}}{1} = \frac{1}{n}$$

Use a method of your choice to solve for n. The cross-product method is shown.

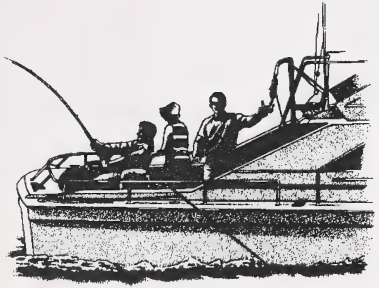
$$\frac{1}{2}n = 1$$
$$n = 2$$

Together they can knit one set in 2 d.

Example 2

A boat has two motors, one large and one small. The large motor uses 10 L of gasoline per hour. The small motor consumes 6 L of gasoline per hour.

If there are 40 L of gasoline on the boat, how long will the gasoline last with the two motors running?



Solution

In 1 h, the small motor will use 6 L of gasoline and the large motor will use 10 L of gasoline.

$$10 + 6 = 16$$

So, the consumption rate will be 16 L/h if both motors are running.

Write a proportion to discover how long it will take to consume 40 L of gasoline if both motors are running.

$$\frac{\text{Gasoline consumed (L)}}{\text{Time (h)}} \qquad \frac{16}{1} = \frac{40}{n}$$

Solve the proportion using a method of your choice. The cross-product method is shown.

$$16n = 40$$
$$n = 2.5$$

The gasoline will last 2.5 h.

Example 3

It takes 10 people 10 d to build 1 house. How long will it take 5 people to build 2 houses?

Solution

Working together, 10 people can build 1 house in 10 d.

Working together, 10 people can build $\frac{1}{10}$ of a house in 1 d.

Working together, 5 people can build $\frac{1}{20}$ of a house in 1 d.

Write a proportion to discover how long it will take 5 people to build 2 houses.

Number of houses / Number of days

$$\frac{\frac{1}{20}}{1} = \frac{2}{n}$$

Use a method of your choice to solve for n. The cross-product method is shown.

$$\frac{1}{20}n = 2$$

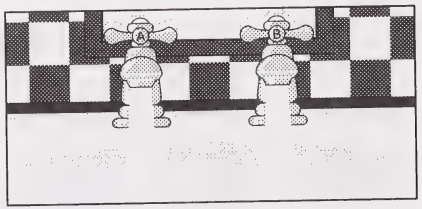
$$n = 40$$

It will take 5 people 40 days to build 2 houses.

Practice Activity 3

1. Susan can paint a room in 3 h. Sherry can paint a room in 4 h. How long will it take to paint the room if they work together?

2. Tap A can fill a sink in 8 min. Tap B can fill a sink in 6 min. How long will take to fill the sink using both taps?

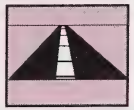


3. 

Todd and Tim can mow the lawn in the park in 2 h if they work together. Todd can do the job alone in 3 h. How long will it take Tim to mow the lawn alone?

4. If 6 hens can lay 24 eggs in 6 days, how many eggs can 3 hens lay in 3 days?

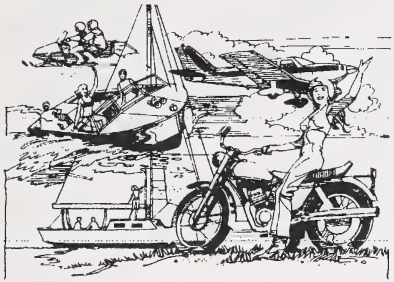
Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will solve more rate problems.

Working Together



Many situations involve the rate of speed. These situations may involve people or animals walking, running, or swimming, or they may involve driving a car, flying a plane, rowing a boat, or travelling in some other kind of vehicle.

Because these situations are encountered frequently, this formula has been developed to show the relationship between the distance travelled (d), the rate of speed (v), and the time travelled (t).

$$d = vt$$

This formula can also be rearranged to solve for speed (v) or time (t).

$$d = vt \qquad d = vt$$

$$\frac{d}{t} = \frac{v\cancel{t}^{1}}{\cancel{t}_{1}} \qquad \frac{d}{v} = \frac{\cancel{v}t^{1}}{\cancel{v}_{1}}$$

$$\frac{d}{t} = v \qquad \frac{d}{v} = t$$

Example 1

A car travels at 50 km/h. How far will the car go in 3 h?

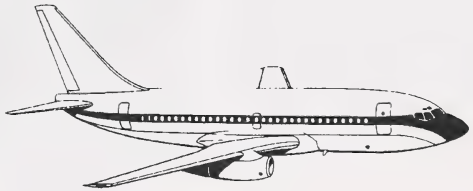
Solution

$$d = vt$$
$$= 50 \times 3$$
$$= 150$$

The car will go 150 km in 3 h.

Example 2

A plane travels at 800 km/h. How long will it take the plane to travel 2400 km?



Solution

There are two ways to solve this problem.

Method 1

Substitute the known values and solve for t.

$$d = vt$$
$$\frac{2400}{800} = \frac{800t}{800}$$
$$3 = t$$

It will take 3 h.

Method 2

Rearrange the formula to solve for t and then substitute the known values.

$$d = vt \qquad \text{So, } t = \frac{d}{v}$$
$$\frac{d}{v} = \frac{vt}{v} \qquad t = \frac{2400}{800}$$
$$\frac{d}{v} = t \qquad = 3$$

It will take 3 h.

Problem Solving

To solve some problems involving distance, time, and speed, you may find the following strategies helpful.

- drawing a diagram
- organizing information in a table

Example 1

An eagle and a hawk leave a tree on which they are perched and fly in opposite directions. The eagle flies at 36 km/h and the hawk flies at 24 km/h. After 3 h, how far apart will they be?

Solution

Make a diagram.

Organize the data in a table.

	d	v	t
Eagle		36	3
Hawk		24	3

Calculate the distance each bird travelled using the formula $d = vt$.

$$d = vt$$
$$= 36 \times 3$$
$$= 108$$

So, the eagle travelled 108 km.

$$d = vt$$
$$= 24 \times 3$$
$$= 72$$

So, the hawk travelled 72 km.

Calculate the distance between the birds.

$$108 + 72 = 180$$

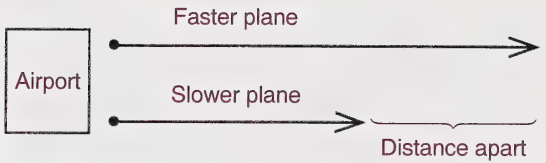
The eagle and the hawk are 180 km apart.

Example 2

Two planes take off from an airport at the same time. They travel in the same direction, but the faster plane flies at a speed of 1020 km/h, and the slower plane flies at 870 km/h. After 2 h, how far apart are they?

Solution

Make a diagram.



Organize the data in a table.

	d	v	t
Faster plane		1020	2
Slower plane		870	2

Calculate the distances flown using the formula $d = vt$.

$$d = vt$$
$$= 1020 \times 2$$
$$= 2040$$

The faster plane flew 2040 km.

$$d = vt$$
$$= 870 \times 2$$
$$= 1740$$

The slower plane flew 1740 km.

Calculate the distance between the planes.

$2040 - 1740 = 300$

The planes are 300 km apart.

Practice Activity 1

1. Martin rode his motorcycle at 80 km/h for 3 h. How far did he travel?

2. A hot-air balloon rose at the speed of 4.2 m/s. How far did it rise in 45 s?

3. Troy lives 2 km from school. If it takes him 45 min to walk to school, what is his walking speed?

4. An elevator descends at the speed of 3.5 m/s. How long will it take the elevator to descend 45 m?

Turn to the Appendix to check your answers.

Working Together

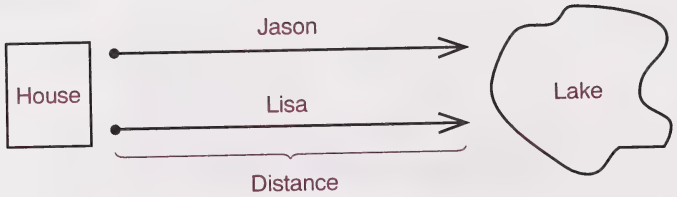
Some problems involving distance, rate, and time are a little more complicated.

Example 1

Jason jogged at a rate of 6 km/h from his house to the lake. One hour later, Lisa followed the same route on her bicylce at a rate of 18 km/h. They reached the lake at the same time. How far was the trip?

Solution

Draw a diagram.



Organize the data in a table.

	d	v	t
Jason		6	
Lisa		18	

Method 1: Guess-Check-Revise Method

Guess the time that it took for each person to travel to the lake. Remember that Lisa travelled for one hour less than Jason.

Guess 1: Lisa travelled for 2 h; Jason travelled for 3 h.

Calculate the distances using the formula $d = vt$.

	d	v	t
Jason	18	6	3
Lisa	36	18	2

Check to see if the guess is correct. Are the distances equal?
No, $18 \neq 36$.

Guess 2: Lisa travelled for 2 h; Jason travelled for 1 h.

Calculate the distances using the formula $d = vt$.

	d	v	t
Jason	6	6	1
Lisa	36	18	2

Check to see if the guess is correct. Are the distances equal?
No, $6 \neq 36$.

Guess 3: Lisa travelled for 0.5 h; Jason travelled for 1.5 h.

Calculate the distances using the formula $d = vt$.

	d	v	t
Jason	9	6	1.5
Lisa	9	18	0.5

Check to see if the guess is correct. Are the distances equal?
Yes, $9 = 9$.

So, the trip was 9 km.

Method 2: Solving an Equation

Let the time Jason travelled be t.
Let the time Lisa travelled be $t - 1$.

Calculate the distances using the formula $d = vt$.

	d	v	t
Jason	$6t$	6	t
Lisa	$18(t-1)$	18	$t-1$

The distance Jason travelled and the distance Lisa travelled are equal. Write an equation that shows this.

$$\begin{aligned} 18(t-1) &= 6t \\ 18t - 18 &= 6t \\ +18 \quad & \quad +18 \\ \hline 18t &= 6t + 18 \\ -6t \quad & \quad -6t \\ \hline 12t &= 18 \\ t &= 1.5 \end{aligned}$$

So, Jason travelled for 1.5 h.

$$\begin{aligned} d &= vt \\ &= 6 \times 1.5 \\ &= 9 \end{aligned}$$

The distance travelled was 9 km.

Example 2

A train travelled through the mountains for 6 h and then travelled through the prairies for 3 h. The total distance travelled was 756 km. If the train went 12 km/h faster through the prairies, what was the speed through the mountains and on the prairies?

Solution

Draw a diagram

Organize the data in a table.

	d	v	t
Mountains			6
Prairies			3

Method 1: Guess-Check-Revise Method

Guess the speed of the train through the mountains and the speed across the prairies. Remember that the train went 12 km/h faster through the prairies.

Guess 1: The train went 95 km/h on the prairies and 83 km/h through the mountains.

Calculate the distances using the formula $d = vt$.

	d	v	t
Mountains	498	83	6
Prairies	285	95	3

Check to see if the guess is correct. Do the distances total 756 km?
No, $498 + 285 \neq 756$.

Guess 2: The train went 90 km/h on the praries and 78 km/h through the mountains.

Calculate the distances using the formula $d = vt$.

	d	v	t
Mountains	468	78	6
Prairies	270	90	3

Check to see if the guess is correct. Do the distances total 756 km?
No, $468 + 270 \neq 756$.

Guess 3: The train went 92 km/h on the prairies and 80 km/h through the mountains.

Calculate the distances using the formula $d = vt$.

	d	v	t
Mountains	480	80	6
Prairies	276	92	3

Check to see if the guess is correct. Do the distances total 756 km?
Yes, $480 + 276 = 756$.

So, the speed through the mountains was 80 km/h and the speed on the prairies was 92 km/h.

Method 2: Using an Equation

Let the speed through the mountains be n.
Let the speed on the prairies be $n + 12$.

Calculate the distances using the formula $d = vt$.

	d	v	t
Mountains	$6n$	n	6
Prairies	$3n + 36$	$n + 12$	3

The total distance travelled is 756 km. Write an equation to show this.

$$\begin{aligned} 6n + 3n + 36 &= 756 \\ 9n + \cancel{36} &= 756 \\ -\cancel{36} \quad &-36 \\ \hline 9n \quad &= 720 \\ n \quad &= 80 \end{aligned}$$

The speed through the mountains was 80 km/h.

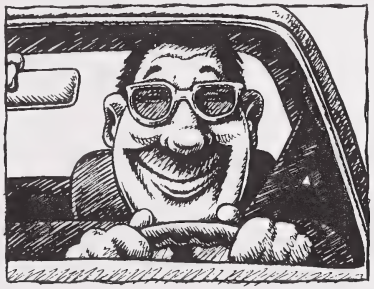
If $n = 80$, then $n + 12 = 80 + 12$
$= 92$

The speed on the prairies was 92 km/h.

Practice Activity 2

Solve the following problems.[1]

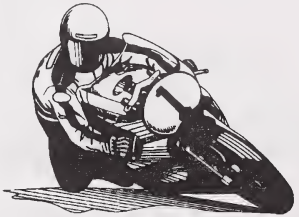
1. Dr. Pepper left Oakville at 9:00 a.m. and drove to Central City at 60 km/h. H. Salt left Oakville at 11:00 a.m. and travelled the same route to Central City. If both men arrived in Central City at 4:00 p.m., at what rate did H. Salt travel?



2. Two trucks left Buck's Trucks and travelled in opposite directions. One truck travelled at a rate of 70 km/h, the other travelled at 80 km/h. After how many hours were the trucks 900 km apart?

3. A truck left Buck's Trucks and travelled north at 80 km/h. One hour later, another truck left Buck's Trucks and travelled south at 60 km/h. How many hours had the first truck travelled when they were 150 km apart?

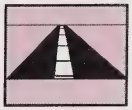
4. Steve McSpoke left home on his bicycle at 8:00 a.m. He was travelling at 18 km/h. At 10:00 a.m., Steve's brother set out after him on a motorcycle, following the same route. The motorcycle travelled at 54 km/h.



   a. How long had Steve travelled when his brother overtook him?
   b. How far had Steve travelled when his brother overtook him?

Turn to the Appendix to check your answers.

[1] 1989 Creative Publications for excerpt from *Algebra With Pizzazz* .



What Lies Ahead

In this section you will learn these skills.

- interpreting percents
- expressing a percent as a decimal number or a fraction
- expressing a fraction as a decimal number or a percent
- expressing a decimal number as a fraction or a percent

Working Together

Many words are formed from the root word *cent*.

- How many centimetres are there in a metre?
- How many cents are there in a dollar?
- How many years are there in a century?

The root word *cent* means "hundred".

The word percent means "out of 100" or "for every 100". A **percent** is a special ratio with 100 as the second term.

90% = 90 : 100

This is read as "90 percent".

Percents are often used in everyday life because this form of ratio is easy to compare. For example partly-skimmed milk can have 1% milkfat or 2% milkfat. Whipping cream has 35% milkfat.

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Video Activity

Watch the video *MATHWAYS: The Percent*. This program provides a good introduction to percents.

Working Together

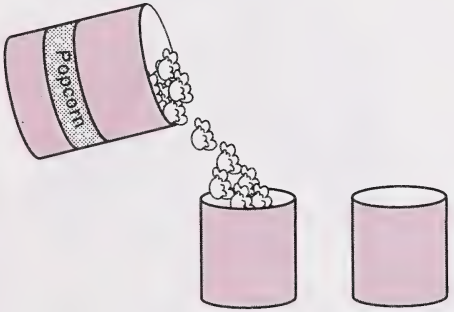
Percents Less Than 1

Notice how a percent less than 1 can be expressed as a fraction or a decimal number.

Example 1

This container of popcorn represents 100% of the popcorn.

The popcorn can be divided into two smaller containers.



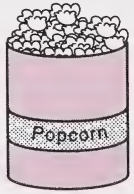
Since $100\% \div 2 = 50\%$, each of the smaller containers has 50% of the popcorn.

$$50\% = \frac{50}{100} \qquad 50\% = \frac{50}{100}$$
$$= \frac{1}{2} \qquad = 0.5$$

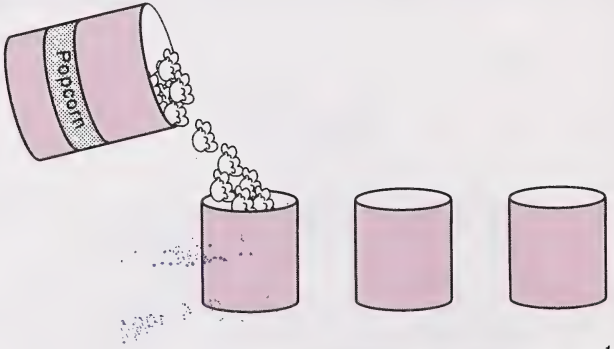
This means that each container has $\frac{1}{2}$ of the popcorn, or 0.5 of the popcorn.

Example 2

This container represents 100% of the popcorn.



The popcorn can be divided into three smaller containers.



Since $100\% \div 3 = 33\frac{1}{3}\%$, each of the smaller containers has $33\frac{1}{3}\%$ of the popcorn.

$$33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100}$$

$$= \frac{\frac{100}{3}}{100}$$

$$= \frac{\overset{1}{\cancel{100}}}{3} \times \frac{1}{\underset{1}{\cancel{100}}}$$

$$= \frac{1}{3}$$

$$33\frac{1}{3}\% = 33.\dot{3}\%$$

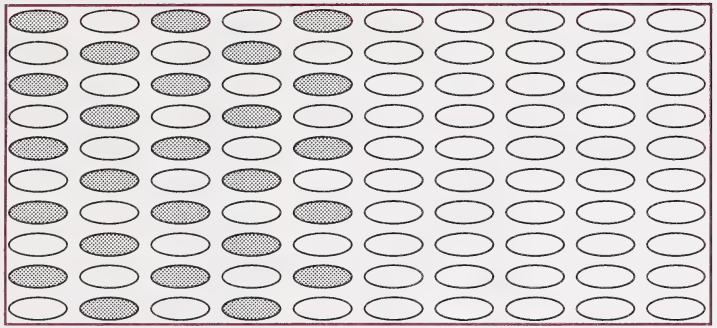
$$= \frac{33.\dot{3}}{100}$$

$$= 0.\dot{3}$$

This means that each container has $\frac{1}{3}$ of the popcorn, or $0.\dot{3}$ of the popcorn.

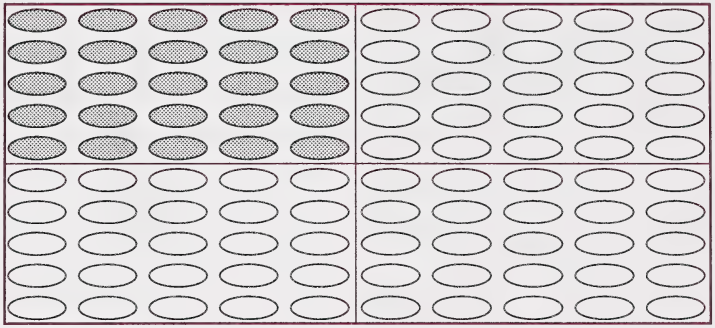
Example 3

25% of the ovals shown are grey.



25% means that 25 ovals out of 100 ovals are grey.

The ovals can be rearranged like this.



$$25\% = \frac{25}{100}$$

$$= \frac{1}{4}$$

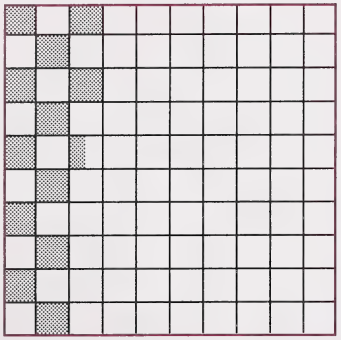
$$25\% = \frac{25}{100}$$

$$= 0.25$$

This means that $\frac{1}{4}$, or 0.25, of the ovals are grey.

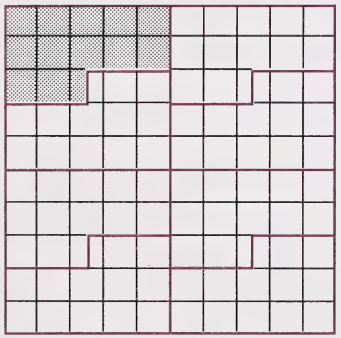
Example 4

$12\frac{1}{2}\%$ of the squares are grey.



$12\frac{1}{2}\%$ means that $12\frac{1}{2}$ squares out of 100 squares are grey.

The squares can be rearranged like this.



$$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100}$$
$$= \frac{25}{2} \div 100$$
$$= \frac{25}{2} \times \frac{1}{100}$$
$$= \frac{1}{8}$$

$$12\frac{1}{2}\% = 12.5\%$$
$$= \frac{12.5}{100}$$
$$= \frac{125}{1000}$$
$$= 0.125$$

This means that $\frac{1}{8}$, or 0.125, of the squares are grey.

Example 5

Seawater is 3.2% salt by mass.

3.2% means that 3.2 parts out of 100 parts of seawater are salt.

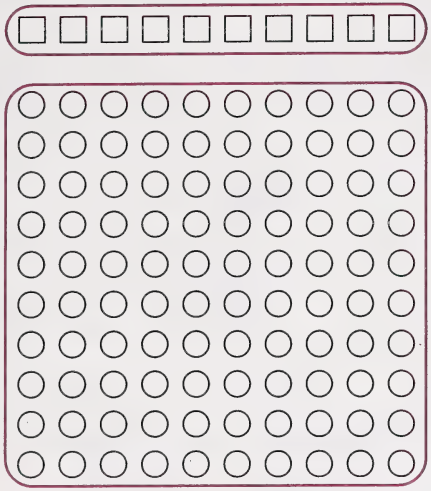
$$3.2\% = \frac{3.2}{100}$$
$$= \frac{32}{1000}$$
$$= \frac{4}{125}$$

$$3.2\% = \frac{3.2}{100}$$
$$= \frac{32}{1000}$$
$$= 0.032$$

This means that $\frac{4}{125}$, or 0.032, of the seawater is salt.

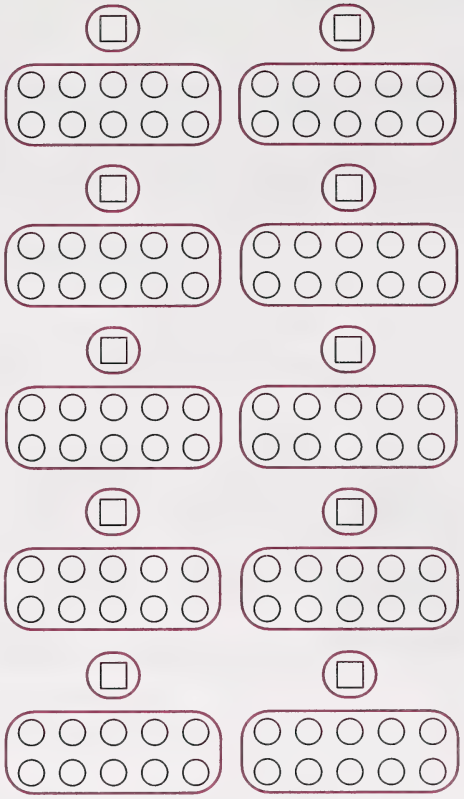
Example 6

The number of squares is 10% of the number of circles.



That means that there are 10 squares for every 100 circles.

By rearranging the circles and squares, you can see that the ratio of the number of squares to the number of circles is 1 to 10.



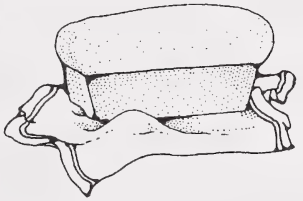
$$10\% = \frac{10}{100} \qquad 10\% = \frac{10}{100}$$
$$= \frac{1}{10} \qquad = 0.1$$

This means that the number of squares is $\frac{1}{10}$, or 0.1, of the number of circles.

Practice Activity 1

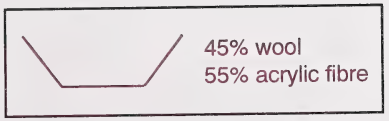
1. A loaf of bread is 60% whole wheat. Express the percent of whole wheat flour in this loaf of bread as each of the following.

   a. a fraction in lowest terms
   b. a decimal number



2. The label on a sweater shows that the sweater is made of wool and acrylic fibre. Express the amount of wool in the sweater as each of the following.

   a. a fraction in lowest terms
   b. a decimal number

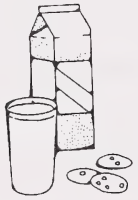


3. A metal alloy is made of zinc, tin, and lead. The amount of tin is 60% of the amount of lead. Express this percent as each of the following.

   a. a fraction in lowest terms
   b. a decimal number

4. Milk is $87\frac{1}{2}\%$ water. Express this percent as each of the following.

   a. a fraction in lowest terms
   b. a decimal number



5. Your body is about $66\frac{2}{3}\%$ water. Express this percent as each of the following.

   a. a fraction in lowest terms
   b. a decimal number

6. In a shipment of telephones, 2.5% of the telephones are defective. Express this percent as each of the following.

   a. a fraction in lowest terms
   b. a decimal number

7. About $\frac{1}{30}\%$ of all water on Earth evaporates each year. Express this percent as each of the following.

   a. a fraction in lowest terms
   b. a decimal number

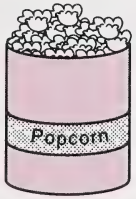
Turn to the Appendix to check your answers.

Percents Greater Than 1

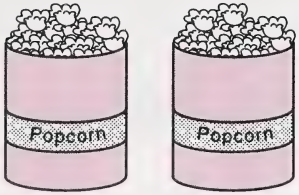
Notice how percents greater than 100% can be written as whole numbers, mixed numbers, or decimal numbers.

Example 1

This container of popcorn represents 100% of the popcorn.



These containers represent 200% of the popcorn.



Express 200% as a whole number.

Solution

$$200\% = \frac{200}{100}$$
$$= 2$$

Example 2

This container represents 100% of the popcorn.

These containers represent 250% of the popcorn.

Express 250% as a mixed number and a decimal number.

Solution

$$250\% = \frac{250}{100}$$
$$= 2\frac{1}{2}$$

$$250\% = \frac{250}{100}$$
$$= 2.5$$

Practice Activity 2

1. This year the hens laid 300% of the eggs they laid last year. Express the percent as a whole number.

2. Joan repaid her mother 110% of the money she borrowed.

   a. Express the percent as a fraction.
   b. Express the percent as a decimal number.

3. Becky is 125% of the height she was two years ago.

   a. Express the percent as a fraction.
   b. Express the percent as a decimal number.

4. The volume of steam is 167 000% of the volume of water. Express this percent as a whole number.

5. An ant can lift a mass that is 5000% of the mass of its own body. Express this percent as a whole number.

Turn to the Appendix to check your answers.

Working Together

Fractions Less Than 1

Notice how a fraction less than 1 can be expressed as a decimal number and as a percent.

Example 1

Mrs. Ronet's expenses are $\frac{4}{5}$ of Mrs. Martinez's expenses. Express the fraction as a decimal number and as a percent.

Solution

You already know that $\frac{4}{5} = 0.8$. So, Mrs. Ronet's expenses are 0.8 of Mrs. Martinez's expenses.

To find the percent, first find the equivalent fraction with 100 as the denominator.

$$\frac{4}{5} \overset{\times 20}{=} \frac{80}{100}$$

Write the equivalent fraction as a percent.

$$\frac{80}{100} = 80\%$$

Mrs. Ronet's expenses are 80% of Mrs. Martinez's expenses.

Example 2

An alloy of copper, zinc, and lead is used to make clock parts. Copper is $\frac{16}{25}$ of the alloy. Express this fraction as a decimal number and as a percent.

Solution

You already know that $\frac{16}{25} = 0.64$. So, copper is 0.64 of the alloy.

To write the fraction as a percent, you must first find an equivalent fraction that has 100 as its denominator.

$$\frac{16}{25} \overset{\times 4}{=} \frac{64}{100}$$

Then write the equivalent fraction as a percent.

$$\frac{64}{100} = 64\%$$

So, 64% of the alloy is copper.

Example 3

In a basketball game, Tom made $\frac{3}{8}$ of the shots he attempted. Express this fraction as a decimal number and as a percent.

Solution

You already know $\frac{3}{8} = 0.375$. So, Tom made 0.375 of the shots he attempted.

To write the fraction as a percent, you must first find an equivalent fraction that has 100 as its denominator.

$$\frac{3}{8} = \frac{n}{100}$$
$$300 = 8n$$
$$\frac{300}{8} = n$$
$$37\frac{1}{2} = n$$

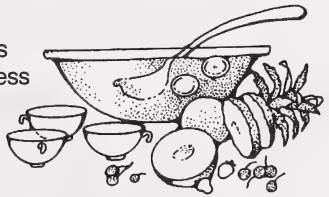
So, Tom made $37\frac{1}{2}\%$, or 37.5%, of the shots that he attempted.

Decimal Numbers Less Than 1

Notice how a decimal number less than 1 can be expressed as a fraction and as a percent.

Example 1

The amount of lemonade in a punch is 0.3 of the amount of ginger ale. Express the decimal number as a fraction and as a percent.



Solution

You already know that $0.3 = \frac{3}{10}$. So, the amount of lemonade is $\frac{3}{10}$ of the amount of punch.

To change the decimal number to a percent, write the decimal number as a fraction.

$$0.3 = \frac{3}{10}$$

Then find the equivalent fraction that has 100 as its denominator.

$$\frac{3}{10} = \frac{30}{100}$$

Finally write the fraction as a percent.

$$\frac{30}{100} = 30\%$$

The amount of lemonade in the punch is 30% of the amount of ginger ale.

Example 2

The 1982 Canadian gold coin is an alloy of gold and silver. 0.92 of the coin is gold. Express the decimal number as a fraction and as a percent.

Solution

You already know that $0.92 = \frac{92}{100}$, or $\frac{23}{25}$. So, $\frac{23}{25}$ of the coin is gold.

To change the decimal number to a percent, first write the decimal number as a fraction with 100 as its denominator.

$$0.92 = \frac{92}{100}$$

Then write the fraction as a percent.

$$\frac{92}{100} = 92\%$$

So, 92% of the coin is gold.

Example 3

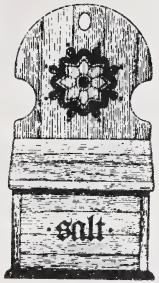


Table salt is composed of sodium chloride and a small quantity of minerals and additives. In fact, 0.9923 of salt is sodium chloride. Express this decimal number as a fraction and as a percent.

Solution

You already know that $0.9923 = \frac{9923}{10\,000}$. So, $\frac{9923}{10\,000}$ of salt is sodium chloride.

To change the decimal number to a percent, first write the decimal number as a fraction with 100 as its denominator.

$$0.9923 = \frac{9923}{10\,000}$$
$$= \frac{99.23}{100}$$
$$= 99.23\%$$

So, 99.23% of salt is sodium chloride.

Practice Activity 3

1. About $\frac{3}{4}$ of the students in the computer class were present. Express the fraction as each of the following.

   a. a decimal number
   b. a percent

2. The amount of fence Bruce painted is $\frac{1}{6}$ of the amount Joan painted. Express the fraction as each of the following.

   a. a decimal number
   b. a percent

3. The baseball player hit safely 0.25 of the times he was up to bat. Express the decimal number as each of the following.

   a. a fraction
   b. a percent

4. Mr. Sabean sowed 0.3 of his field in barley. Express the decimal number as each of the following.

   a. a fraction
   b. a percent

Turn to the Appendix to check your answers.

Whole Numbers and Mixed Numbers

Notice how whole numbers and mixed numbers can be expressed as percents.

Example 1

A penguin egg hatches in 5 times the amount of time in which a sparrow egg hatches. Express this as a percent.

Solution

$$5 = \frac{5}{1}$$
$$= \frac{500}{100}$$
$$= 500\%$$

The time for a penguin egg to hatch is 500% of the time for a sparrow egg to hatch.

Example 2

An ostrich's mass is about $7\frac{1}{2}$ times that of the heaviest flying bird. Express this as a percent.

Solution

$$7\frac{1}{2} = \frac{15}{2}$$
$$= \frac{750}{100}$$
$$= 750\%$$

The mass of an ostrich is 750% of the mass of the heaviest flying bird.

Example 3

The speed at which the canvas-back duck can fly is 2.3 times the speed at which an ostrich can run. Express this as a percent.

Solution

$$2.3 = \frac{2.3}{1}$$
$$= \frac{230}{100}$$
$$= 230\%$$

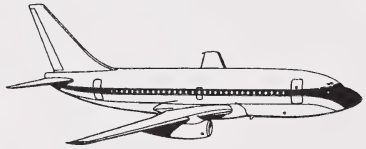
The speed at which a canvas-back duck can fly is 230% of the speed at which an ostrich can run.

Example 4

Some jet planes travel at a speed that is about 6 times the speed of the spine-tail swift. Express this as a percent.

Solution

$$6 = \frac{6}{1}$$
$$= \frac{600}{100}$$
$$= 600\%$$



Some jet planes fly at 600% of the speed of the spine-tail swift.

Practice Activity 4

1. A supersonic jet can fly at 2.2 times the speed of sound. Express this as a percent.

2. A rocket can fly at $5\frac{3}{4}$ times the speed of sound. Express this as a percent.

3. The length of a nail is $2\frac{1}{3}$ times the length of a paper clip. Express this as a percent.

4. The length of a pencil is $1\frac{2}{3}$ times the length of a nail. Express this as a percent.

5. The mass of an ostrich's egg is 1 000 000 times the mass of a hummingbird's egg. Express this as a percent.

Turn to the Appendix to check your answers.

Working Together

Mental Computation

If you remember some percent equivalents, you can figure out others mentally.

You should first memorize these relationships.

$1\% = \frac{1}{100}$ $\quad 10\% = \frac{1}{10}$ $\quad 25\% = \frac{1}{4}$

$33\frac{1}{3}\% = \frac{1}{3}$ $\quad 50\% = \frac{1}{2}$

Gradually you will remember other percent equivalents.

Example 1

Express $\frac{3}{10}$ as a percent.

Solution

Think $\frac{3}{10} = 3 \times \frac{1}{10}$

$= 3 \times 10\%$

$= 30\%$

Example 2

Express $\frac{1}{12}$ as a percent.

Solution

Think $\frac{1}{12} = \frac{1}{3} \times \frac{1}{4}$

$= \frac{1}{3} \times 25\%$

$= 8\frac{1}{3}\%$

Example 3

Express $\frac{3}{8}$ as a percent.

Solution

Think $\frac{1}{8} = \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{2} \times 25\%$

$= 12\frac{1}{2}\%$

Think $\frac{3}{8} = 3 \times \frac{1}{8}$

$= 3 \times 12\frac{1}{2}\%$

$= 37\frac{1}{2}\%$

Find the percent equivalent to $\frac{1}{8}$ and multiply by 3.

Practice Activity 5

1. Use mental computation to express each of the following as a percent.

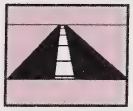
   a. $\frac{7}{10}$ b. $\frac{9}{100}$ c. $\frac{1}{20}$

   d. $\frac{1}{200}$ e. $\frac{1}{16}$ f. $\frac{1}{5}$

2. Use mental computation to express each of the following as a percent.

   a. $\frac{3}{20}$ b. $\frac{7}{200}$ c. $\frac{5}{8}$ d. $\frac{4}{5}$

Turn to the Appendix to check your answers.



What Lies Ahead

In this section you will solve percent problems.

Working Together

In this course you have learned that there is often more than one way to solve a problem. In this section you will learn to solve percent problems in two ways – using proportions and writing equations.

Example 1

The volleyball team sold $375 worth of chocolate bars. If they kept 20% of the sales, how much did they keep for themselves?

Solution

There are two methods to solve this problem.

Method 1

Write an equation.

20% of $375 is _____ .

$20\% \times 375 = n$

$0.2 \times 375 = n$

Solve the equation.

$0.2 \times 375 = n$

$75 = n$

The team kept $75.

Note: Many calculators have a [%] key. If you use the [%] key, you do not need to change the percent to a decimal number or press the [=] key.

If your calculator has a [%] key, enter the number to be multiplied first and the percent key last.

Key Press	Display
3 7 5 × 2 0 %	75

Method 2

Write a proportion.

Amount kept / Amount sold

$$\frac{n}{375} = \frac{20}{100}$$

Solve the proportion using a method of your choice. The cross-product method is shown.

$$100n = 7500$$
$$n = 75$$

The team kept $75.

Example 2

Advance tickets to a rock concert are sold for $20 each. This is 80% of the price at the door. How much will a ticket cost at the door?

Solution

There are two methods to solve the problem.

Method 1

Write an equation.

Price of tickets at door — Price of advance tickets

80% of _____ is $20.

$$80\% \times n = 20$$
$$0.8n = 20$$

Solve the equation.

$$\frac{0.8n}{0.8} = \frac{20}{0.8}$$
$$n = 25$$

The price of tickets at the door is $25.

Method 2

Price of advance tickets / Cost of tickets at door

$$\frac{20}{n} = \frac{80}{100}$$

Solve the proportion using a method of your choice. The cross-product method is shown.

$$2000 = 80n$$
$$25 = n$$

The price of tickets at the door is $25.

Example 3

Dan earns a commission of $7200 on a house that he sells. If he sells the house for $120 000, what percent of commission does he earn?

Solution

There are two ways to solve the problem.

Method 1

Write an equation.

Selling price Commission

↓ ↓

_____ % of 120 000 is 72 000.

$$n\% \times 120\,000 = 7200$$

$$\frac{n}{100} \times 120\,000 = 7200$$

Solve the equation.

$$\frac{n}{\cancel{100}_{1}} \times \cancel{120\,000}^{1200} = 7200$$

$$\frac{1200n}{1200} = \frac{7200}{1200}$$

$$n = 6$$

So, the commission earned is 6%.

Method 2

Write a proportion.

Commission / Selling price

$$\frac{7200}{120\,000} = \frac{n}{100}$$

Solve the proportion using a method of your choice. The cross-product method is shown.

$$720\,000 = 120\,000n$$

$$6 = n$$

So, the commission earned is 6%.

Practice Activity 1

Solve the following problems.[1]

1. Profits of Calculess Corporation this year were 140% of the profits last year. If profits last year were $5200, what were the profits this year?

2. Klutz got 10 out of 16 problems on an algebra test correct. What percent were correct?

3. A steel cable expands 0.2% of its length when its temperature is increased 100°C. How much longer will a 750-m cable become with this increase in temperature?

4. A team won 13 games, lost 15 games, and tied 2 games. What percent of its games did the team win?

5. In a magazine drive a school keeps 40% of all sales dollars. How many dollars worth of magazines must be sold for the school to earn $5000?

6. A real estate broker earns $2\frac{1}{2}$% of her sales as a commission. How many dollars in sales does she need in order to earn a commission of $1000?

7. Elmo Buckets made 54 out of 80 free throws. What percent did he miss?

Turn to the Appendix to check your answers.

Working Together

One of the most common applications of percent can be found when you go shopping.

Discount

Almost every day you encounter the term **discount**. Discount can be defined as the amount by which the regular price is reduced. What does this mean in dollars and cents?

Example

What is the value of the discount on a pair of shoes that are regularly priced at $179.99 but that are now selling at a 25% discount?

[1] 1989 Creative Publications for excerpts from *Pre-Algebra with Pizzazz.*

Solution

$$25\% \text{ of } 179.99 = 0.25 \times 179.99$$
$$= 44.9975$$
$$= 45.00$$

The discount is $45.

Note: Store prices are always rounded up.

Sale Price

The **sale price** is the difference between the regular price and the discount.

Example

A store is selling video games at 5% discount. If a video game is regularly priced at $40, what is its price with the 5% discount?

Solution

$$5\% \text{ of } 40 = 0.05 \times 40$$
$$= 2$$

$$5\% = \frac{5}{100}$$
$$= 0.05$$

The store is willing to take $2 off the regular price.

$$\text{Sale price} = \text{Regular price} - \text{Discount}$$
$$= 40 - 2$$
$$= 38$$

The sale price of the video game is $38.

Sales Tax

In Canada a **goods and services tax (GST)** of 7% is added on to the price of most items you buy.

In all provinces except Alberta, a **provincial sales tax (PST)** is also charged on these items. The provincial sales taxes vary from province to province.

Province	Sales Tax
Newfoundland	12%
New Brunswick	11%
Nova Scotia	10%
Prince Edward Island	10%
Quebec	9%
Ontario	8%
Manitoba	7%
Saskatchewan	7%
British Columbia	6%
Alberta	0%

Example

What would a pair of skis cost a customer in Alberta, Newfoundland, and British Columbia if the selling price is $418.98?

Solution

Alberta

In Alberta there is no provincial sales tax. Only the goods and services tax (GST) is calculated.

$$\begin{aligned} 7\% \text{ of } 418.98 &= 0.07 \times 418.98 \\ &= 29.3286 \\ &\doteq 29.33 \end{aligned}$$

The GST on the skis is $29.33.

$$418.98 + 29.33 = 448.31$$

So, the cost of the skis to a customer in Alberta is $448.31.

Newfoundland

In the Atlantic provinces and Quebec, the GST is calculated first and added to the selling price. Then the provincial sales tax (PST) is calculated.

$$\begin{aligned} 7\% \text{ of } 418.98 &= 0.17 \times 418.98 \\ &= 29.3286 \\ &\doteq 29.33 \end{aligned}$$

The GST on the skis is $29.33.

$$418.98 + 29.33 = 448.31$$

The cost before the PST is added is $448.31.

$$\begin{aligned} 12\% \text{ of } 448.31. &= 0.12 \times 448.31 \\ &= 53.7972 \\ &\doteq 53.80 \end{aligned}$$

The PST on the skis is $53.80.

$$448.31 + 53.80 = 502.11$$

So, the cost of the skis to a customer in Newfoundland is $502.11.

British Columbia

In the provinces other than the Atlantic provinces, the GST and PST are calculated side by side.

$$\begin{aligned} (7\% + 6\%) \text{ of } 418.98 &= 13\% \text{ of } 418.98 \\ &= 0.13 \times 418.98 \\ &= 54.4674 \\ &= 54.47 \end{aligned}$$

The total tax (GST and PST) on the skis is $54.47.

$$418.98 + 54.47 = 473.45$$

So, the cost of the skis to a customer in British Columbia is $473.45.

Practice Activity 2

1. Calculate the sale price for each of the following items.

   a. A TV that regularly costs $950 is on sale at a discount of 25%.

   b. A microwave oven that regularly costs $249.99 is on sale at a discount of 20% off.

2. Calculate the cost, including the GST and PST, for each of the following items.

   a. a skateboard that costs $89.99 in British Columbia
   b. a watch that costs $17.50 in Alberta
   c. running shoes that cost $49.95 in Nova Scotia

Turn to the Appendix to check your answers.

Working Together

When solving problems that involve discount or sales tax, alternative methods can be used.

Example 1

A tire that has a regular price of $60 is on sale for 30% off. What is the sale price of this tire?

Solution

30% off means that you pay 70% of the regular price.

$$100\% - 30\% = 70\%$$

To find the sale price, calculate the value of 70% of the regular price.

$$70\% \text{ of } 60 = 0.70 \times 60$$
$$= 42$$

The sale price of the item is $42.

Note: It is easy to calculate the sale price using a calculator with this method. If your calculator has a [%] key, enter the number to be multiplied first and the percent key last.

Key Press	Display
6 0 × 7 0 %	42

The sale price is $42.

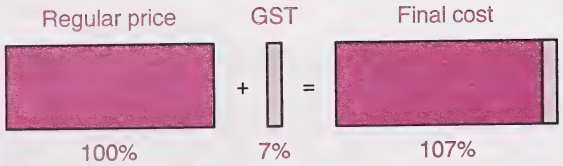
Example 2

The GST is 7%. Find the total cost of an item that sells for $18 in Alberta.

Solution

An extra charge of 7% means that you pay 107% of the regular price.

$$100\% + 7\% = 107\%$$



To find the final cost, calculate 107% of the regular price.

$$107\% \text{ of } 18 = 1.07 \times 18$$
$$= 19.26$$

The final cost of the item is $19.26.

Note: It is easy to use a calculator with this method. Remember to enter the number to be multiplied first and the percent last.

Key Press	Display
1 8 × 1 0 7 %	19.26

The final cost of the item would be $19.26.

Practice Activity 3

1. A department store is offering a 30% discount on regularly priced items. Find the sale price of each of these items.

   a. a coat that is regularly priced at $35.
   b. shoes that are regularly priced at $89.99

2. A car is priced at $18 500 in Alberta and the GST is 7%. What is the final cost of the car?

3. A sweater sells for $39 in Manitoba, the GST is 7%, and the PST is 7%. What is the final cost of the sweater?

Turn to the Appendix to check your answers.

Working Together

Now that you have practised some routine percent problems, see if you can solve the following non-routine problems. Try to use some of the problem-solving skills you have learned in this course.

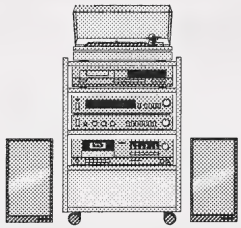
Practice Activity 4

1. A farmer had a herd of cattle which he divided among his four children. He gave 50% of the cattle to the first child, 25% of the cattle to the second child, 20% of the cattle to the third child, and seven cattle to the fourth child. How many cattle did the farmer have?

2. A jacket is reduced by $37.50, which is a discount of 25%.

a. Find the regular price.
b. Find the sales price.

3. A merchant put a stereo set on sale at 10% discount. After two weeks, the merchant reduced the sale price by another 10%. The next week the merchant reduced the sale price by 10% again. Finally, at the end of the month, the merchant took off another 10%. A customer then bought the stereo for $852.93 (This price did not include GST.). What was the original price of the stereo?



Turn to the Appendix to check your answers.

USING ESTIMATION AND MENTAL COMPUTATION TO SOLVE PERCENT PROBLEMS

What Lies Ahead

In this section you will learn these skills.

- estimating a percent of a number
- mentally computing a percent of a number

Working Together

Estimation

Sometimes in everyday life you only need to estimate the percent of a number. At other times you need to find an exact answer.

Even when an exact answer is required, you should make an estimate first. An estimate will provide a "ballpark answer" with which to compare the calculated answer.

Example 1

Mr. Allan decides to leave his waiter 15% of the food bill as a tip. If the food bill was $67.98, how much should Mr. Allan leave as a tip?

Solution

An estimate is sufficient in this situation. Remember that estimation is done mentally.

There are several ways to estimate the amount of the tip. Three methods are shown: using rounding, using betweenness, and using front-end digits.

Method 1

To make an estimate, you can use rounding.

Think 15% of $67.98 \doteq 15\%$ of 70

$\doteq 0.15 \times 70$

$\doteq 70 \times 0.15$

$\doteq 10.50$

Mr. Allan's tip for the waiter should be about $10.50.

Method 2

You can use betweenness to make an estimate. Round down and round up to find a range.

Think 15% of $67.98 \doteq 10\%$ of 60

$\doteq 0.1 \times 60$

$\doteq 6.00$

Think 15% of $67.98 \doteq 20\%$ of 70

$\doteq 0.2 \times 70$

$\doteq 14.00$

Mr. Allan's tip for the waiter should be between $6.00 and $14.00.

Method 3

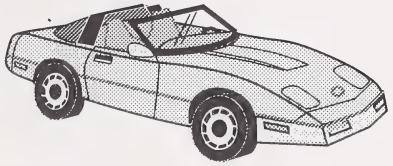
You can use front-end digits to make an estimate. Remember that the front-end digits method gives a low estimate.

Think 15% of 67.98 $\doteq$ 10% of 60
$\doteq 0.1 \times 60$
$\doteq 6.00$

Mr. Allan's tip for the waiter should be over $6.

Example 2

Mr. Kurtz sells cars. He earns a 5% commision on his sales. If Mr. Kurtz sells a car for $18 050, how much does he earn on this one car alone?



Solution

An exact answer is required. An estimate should be made first to ensure that the answer is reasonable.

Estimates can be made in several ways.

Method 1

To make an estimate, you can use rounding.

Think 5% of 18 050 $\doteq$ 5% of 20 000
$\doteq 0.05 \times 20\,000$
$\doteq 1000$

Mr. Kurtz would earn about $1000 on this car alone.

Method 2

You can also use front-end digits to make an estimate.

Remember that the front-end digits method gives a low estimate.

Think 5% of 18 050 $\doteq$ 5% of 10 000
$\doteq 0.05 \times 10\,000$
$\doteq 500$

Mr. Kurtz would earn over $500 on this car alone.

Method 3

You could use betweenness to make an estimate. Remember to round down and then round up.

Think 5% of 18 050 $\doteq$ 5% of 18 000
$\doteq 0.05 \times 18\,000$
$\doteq 900$

Think 5% of 18 050 $\doteq$ 5% of 19 000
$\doteq 0.05 \times 19\,000$
$\doteq 950$

Mr. Kurtz earns between \$900 and \$950 on this car alone.

Find the exact answer and compare the answer to the estimate to decide if it's reasonable.

5% of 18 050 $= 0.05 \times 18\,050$
$= 902.50$

Mr. Kurtz earns \$902.50 in commission on this car alone.

Note: You may use a calculator to find the exact answer. Remember to enter the number first and the percent second. It is not necessary to press [=].

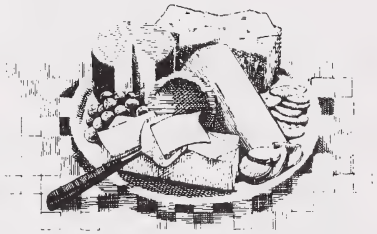
Key Press	Display
[1] [8] [0] [5] [0] [×] [5] [%]	902.5

Using the Decimal Number Form

Sometimes when you are estimating, it is easier to use the decimal number equivalent of the percent.

Example

A package of cheese has 23% milk fat. If the package holds 275 g of cheese, about how much milk fat is there in the package?



Solution

The amount of milk fat is estimated like this.

"Of" means "multiply".

Think 23% of 275 $= 23\% \times 275$
$\doteq 0.2 \times 300$
$\doteq 60$

There are about 60 g of milk fat in 275 g of cheese.

Using the Fraction Form

Sometime when you are estimating, it is easier to use the fraction form of the percent.

Example

Andy scored 73% on a test. If there were 40 questions on the test, about how many questions did Andy get right?

Solution

The number of questions Andy got right is estimated like this.

Think 73% of 40 $\doteq$ 75% of 40

$$\doteq \frac{3}{4} \times 40$$

$$\doteq 30$$

Andy got about 30 questions right out of a total of 40 questions.

Practice Activity 1

Print Alternative

1. Estimate an answer for each of the following.

a. 98% of 680 **b.** 26% of 399 **c.** 32% of 180
d. 49% of 105 **e.** 11% of 58

Computer Alternative

2. Do the program "Making Sense of Percents" on Disk B of *MAC 6*. Read the instructions in the folder with the disk before using the program.

Turn to the Appendix to check your answers.

Working Together

Mental Computation

So far in this module you have learned to calculate a percent of a number using paper-and-pencil methods or using a calculator. Now you will learn to compute a percent of a number mentally.

Using the Decimal Number Form

Sometimes when you are computing mentally, it is easier to work with the decimal number form of the percent.

Example 1

What is 10% of $31.20?

Solution

Think 10% of 31.20 = 0.1 × 31.20
= 3.12

31.20 ← 2 decimal places
× 0.1 ← 1 decimal place
3.12 ← 3 decimal places

Example 2

What is 1% of $31.20?

Solution

Think 1% of 31.20 = 0.01 × 31.20
= 0.31

31.20 ← 2 decimal places
× 0.01 ← 2 decimal places
0.3120 ← 4 decimal places

Example 3

What is 0.1% of $31.20?

Solution

Think 0.1% of 31.20 = 0.001 × 31.20
= 0.03

31.20 ← 2 decimal places
× 0.001 ← 3 decimal places
0.03120 ← 5 decimal places

Example 4

What is 20% of 31.20?

Solution

Think 10% of 31.20 = 0.01 × 31.20
= 3.12

Find 10% and double this amount.

20% of 31.20 = 6.24

Example 5

What is 30% of $31.20?

Solution

Think 10% of 31.20 = 3.12
30% of 31.20 = 9.36

Find 10% and triple this amount.

Example 6

What is 2% of $31.20?

Solution

Think 1% of $31.20 = 0.01 \times 31.20$
$= 0.31$

2% of $31.20 = 0.62$

Find 1% and double this amount.

Example 7

What is 5% of $31.20?

Solution

Think 10% of $31.20 = 0.1 \times 31.20$
$= 3.12$

5% of $31.20 = 1.56$

Find 10% and divide this amount by 2.

Using the Fraction Form

Sometimes when you are computing mentally, it is easier to work with the fraction form of the percent.

Example 1

What is 25% of 40?

Solution

Think 25% of $40 = \frac{1}{4} \times 40$
$= 10$

Multiplying by $\frac{1}{4}$ is the same as dividing by 4.

Example 2

What $33\frac{1}{3}\%$ of 60?

Solution

Think $33\frac{1}{3}\%$ of $60 = \frac{1}{3} \times 60$
$= 20$

Multiplying by $\frac{1}{3}$ is the same as dividing by 3.

Example 3

What is $66\frac{2}{3}\%$ of 60?

Solution

Think $33\frac{1}{3}\%$ of $60 = \frac{1}{3} \times 60$
$= 20$

$66\frac{2}{3}$ of $60 = \frac{2}{3} \times 60$
$= 40$

Find $33\frac{1}{3}\%$ of 60 and multiply by 2.

Practice Activity 2

1. Complete the following using mental computation.

   a. 10% of 48
   b. 10% of 7.25
   c. 1% of 73
   d. 1% of 5.23
   e. 20% of 35
   f. 20% of $9.20
   g. 50% of 46
   h. 50% of $8.20
   i. $33\frac{1}{3}$% of 123
   j. $33\frac{1}{3}$% of $4.50
   k. $66\frac{2}{3}$% of 30
   l. $66\frac{2}{3}$% of $3.60
   m. 25% of 88
   n. 25% of $12.80
   o. 75% of 80
   p. $12\frac{1}{2}$% of $16.80

You may use a calculator for Questions 2, 5, and 8.

2. Compute each of the following.

   a. 18% of 50
   b. 50% of 18
   c. 88% of 25
   d. 25% of 88

3. What pattern did you notice in Question 2?

4. Apply the pattern you discovered in Question 3 to compute each of the following mentally.

   a. 26% of 50
   b. 84% of 25
   c. 55% of 20

5. Compute each of the following.

   a. 45% of 60
   b. 20% of 60 + 25% of 60
   c. 60% of 42
   d. 50% of 42 + 10% of 42
   e. 19% of 24
   f. 20% of 24 – 1% of 24

6. What pattern did you notice in Question 5?

7. Apply the pattern you discovered in Question 6 to compute each of the following mentally.

   a. 35% of 80
   b. 31% of 60
   c. 79% of 50
   d. 11% of 40

8. Compute each of the following.

   a. 25% of 48
   b. 50% of 24
   c. 15% of 70
   d. 30% of 35
   e. 90% of 50
   f. 45% of 100

9. What pattern did you discover in Question 8?

10. Apply the pattern you discovered in Question 9 to calculate each of the following mentally.

    a. 15% of 32
    b. 50% of 36
    c. 20% of 50
    d. 36% of 200
    e. 24% of 20
    f. 35% of 60

Turn to the Appendix to check your answers.

What Lies Ahead

In this section you will learn to calculate simple interest and solve problems involving interest.

Working Together

Interest is the amount paid for the use of money. For example, the bank pays you interest when you keep money in a savings account. You pay the bank interest when you borrow money.

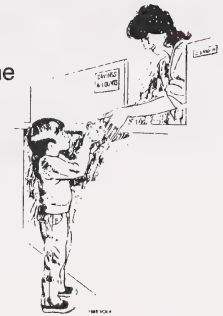
The money which is invested or borrowed is the **principal**. Simple interest can be calculated by the following formula where I is the interest in dollars, P is the principal in dollars, r is the rate of interest per year expressed as a decimal number, and t is the length of time in years.

$$I = Prt$$

Example 1

Randy deposited \$30 in a savings account for one year. If the bank pays him simple interest of $5\frac{1}{2}$%/a on his deposit, how much interest did his savings earn?

Note: The symbol for year is a.



Solution

Think $5\frac{1}{2}\% = 5.5\%$

$= 0.055$

$I = Prt$

$= 30 \times 0.055 \times 1$

$= 1.65$

Randy earned \$1.65 in interest.

Example 2

Mrs. Marston borrowed \$4000 from the bank for one year. If the bank charged her simple interest of $12\frac{3}{4}$%/a on the loan, how much interest did Mrs. Marston pay?

Solution

Think $12\frac{3}{4}\% = 12.75\%$

$= 0.1275$

$I = Prt$

$= 4000 \times 0.1275 \times 1$

$= 510$

Mrs. Marston paid \$510 in interest.

Practice Activity 1

Calculate the interest for each of the following situations.

1. \$1500 is invested at 6% for 3 a.

2. \$12 000 is invested at $12\frac{1}{2}\%$ for 2 a.

3. \$7500 is borrowed at 8% for $1\frac{1}{2}$ a.

Turn to the Appendix to check your answers.

Working Together

Since the interest rate is expressed in %/a, you must express the time in years. Therefore, if the time is given in months or days, you must first change the time to a fraction of a year.

If interest is given as a monthly rate or a daily rate, you should also change it to an annual rate.

Example 1

Dale Winters deposited \$2400.00 in a savings account. The bank pays him simple interest of 6%/a on his deposit. Calculate the interest he earned in 7 mo.

Note: The symbol for month is mo.

Solution

Think $7 \text{ mo} = \frac{7}{12} \text{ a}$

$6\% = 0.06$

$$I = Prt$$
$$= 2400 \times 0.06 \times \frac{7}{12}$$
$$= \$84$$

Dale earned \$84 in interest.

Example 2

A department store credit card charges interest of $2\frac{1}{2}\%$ per month. Ruth owes \$300 for 8 months on her credit card. Calculate the interest she pays.

Solution

Think $2\frac{1}{2}\%/\text{mo} = 30\%/\text{a}$

$30\% = 0.3$

$12 \times 2\frac{1}{2} = 30$

$8 \text{ mo} = \frac{8}{12} \text{ a, or } \frac{2}{3} \text{ a}$

$$I = Prt$$
$$= 300 \times 0.3 \times \frac{2}{3}$$
$$= 60$$

Ruth pays $60 in interest.

Practice Activity 2

Find the simplest interest in each of the following situations.

1. $3000 is borrowed at 5%/a for 3 mo.
2. $6500 is invested at $11\frac{1}{2}$%/a for 90 d.
3. $450 is borrowed at $1\frac{1}{4}$%/mo for 2 a.
4. $3900 is borrowed at 2%/mo for 5 mo.

Turn to the Appendix to check your answers.

Working Together

In this course you learned that problems can be solved in several ways. Problems involving simple interest can be solved by the guess-check-revise method or by using algebra.

Example

Mrs. Pachkowski invested $15 000. Part of this amount was invested at 12% and the rest was invested at $10\frac{1}{2}$%. How much was invested at each rate if the annual interest was $1732.50?

Method 1

Use the guess-check-revise method.

	Amount Invested at 12%	Amount Invested at $10\frac{1}{2}$%	Test
Guess 1	8000	7000	$960 + 735 \neq 1732.50$
Guess 2	10 000	5000	$1200 + 525 \neq 1732.50$
Guess 3	10 500	4500	$1260 + 472.50 = 1732.50$

Mrs. Pachkowski invested $10 500 at 12% and $4500 at $10\frac{1}{2}$%.

Method 2: Using Algebra

Let the amount invested at 12% be n.

Let the amount invested at $10\frac{1}{2}\%$ be $15\,000 - n$.

12% = 0.12,
$10\frac{1}{2}\% = 0.105$

$$0.12n + 0.105(15\,000 - n) = 1732.50$$
$$0.12n + 1575 - 0.105n = 1732.50$$
$$0.015n + 1575 = 1732.50$$
$$-1575 \quad -1575$$
$$0.015n = 157.50$$
$$n = 10\,500$$

If $n = 10\,500$, then $15\,000 - n = 15\,000 - 10\,500$
$= 4500$

Mrs. Pachkowski invested \$10 500 at 12% and \$4500 at $10\frac{1}{2}\%$.

Practice Activity 3

Solve the following problems.[1]

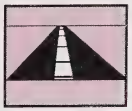
1. Ms. Twinkle invested part of her savings at 6% and the rest at 9%. The amount at 9% was twice the amount at 6%. If her total interest after one year was \$72, find the amount invested at each rate.

2. A scholarship fund raided \$7000 in contributions. Part was invested in bonds paying 6% interest, and the rest was invested in bank certificates paying $8\frac{1}{2}\%$. If the total annual interest is \$520, find the amount invested at each rate.

3. Rockjaw invested part of his savings at 7% and the rest at 13%. The amount at 7% was \$200 more than the amount at 13%. If his total interest after one year was \$84, find the amount invested at each rate.

Turn to the Appendix to check your answers.

[1] 1989 Creative Publications for the excerpt from *Pre-Algebra with Pizzazz.*



What Lies Ahead

In this section you will learn these skills.

- interpreting a scale drawing
- making a scale drawing

Working Together

In Module 1 you learned that sketches can be helpful in solving problems. Sketches are not drawn accurately; parts of the sketch are not proportional to the original scale drawings. A **scale drawing** is a drawing in which all lengths are reductions or enlargements of actual distances.

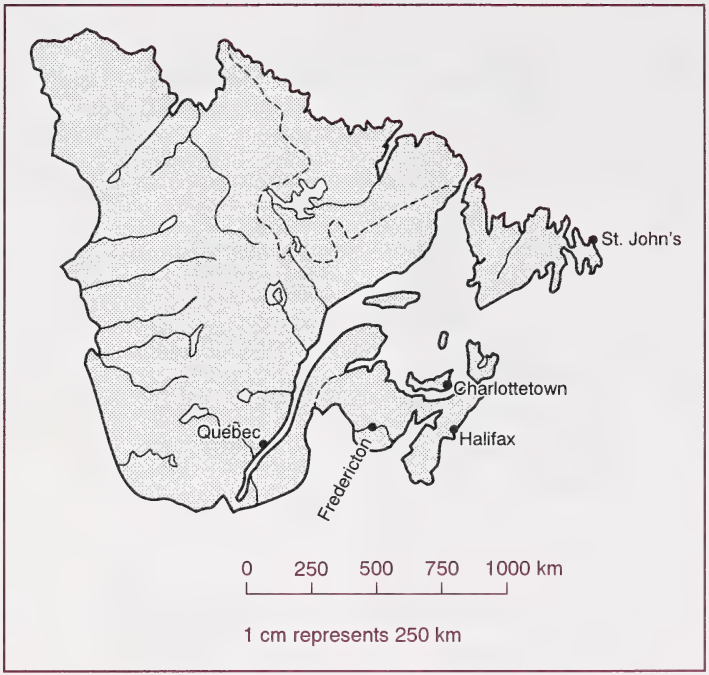
Video Activity

Watch the video *SOLVE IT: Scale Drawing and Models.* You will learn to decide when a scale drawing or model is helpful, how to determine how much bigger or smaller an object is in comparison to its scale drawing or model, and how to decide which unit to use in a given situation.

Working Together

It is useful to know how to use the scale on maps and other scale drawings to calculate distances.

Example



What is the distance from St. John's to Fredericton?

Solution

Measure the straight-line distance on the map between these two cities and then use the scale to write a proportion.

$$\frac{\text{Distance on drawing (cm)}}{\text{Actual distance (km)}} \qquad \frac{4.5}{n} = \frac{1}{250}$$

Solve the equation using a method of your choice. The cross-product method is shown.

$$4.5 \times 250 = n$$
$$1125 = n$$

So, it is 1125 km from St. John's to Fredericton.

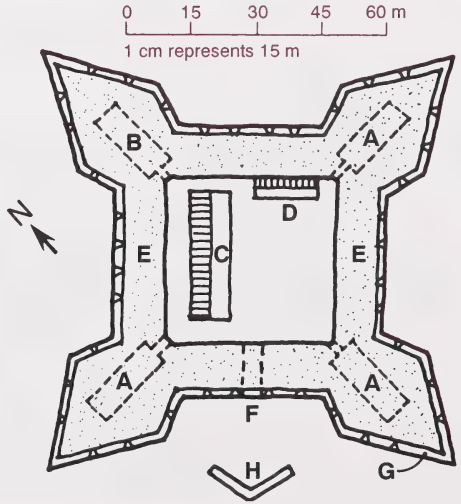
Practice Activity 1

1. Use the following scale drawing to calculate the actual length and width of these structures.

a. dwelling house
b. offices

Prince of Wales Fort About 1770

A Storehouses
B Magazine
C Dwelling house
D Offices
E Earth wall
F Gate
G Stone parapet
H Ravelin to protect gate



2. Use the following map to calculate the actual straight-line distances between these cities.

a. Charlottetown and Halifax
b. Quebec City and St. John's

Turn to the Appendix to check your answers.

Working Together

Some photcopying machines allow you to make copies that are reductions or enlargements of the original. On one such machine you can make copies that are 64% to 156% of the original size.

Example 1

Helga has a drawing that is 10 cm by 12 cm. If she photocopies the drawing at a setting of 85%, what will be the dimensions of the copy?

Solution

Caclulate the length.

$$85\% \text{ of } 12 = 0.85 \times 12$$
$$= 10.2$$

The length is 10.2 cm.

Calculate the width.

$$85\% \text{ of } 10 = 0.85 \times 10$$
$$= 8.5$$

The width is 8.5 cm.

So, the dimensions of the copy will be 8.5 cm by 10.2 cm.

Practice Activity 2

1. Rachel has a drawing that is 6 cm by 4 cm. If she photocopies the drawings at a setting of 130%, what will be the dimensions of the photocopy?

2. Helmut has a drawing that is 8 cm long.

   a. If he wants to make a copy that is 4 cm long, what setting should he select on the photocopier?

   b. If he wants to make a drawing that is 10 cm long, what setting should he select?

Turn to the Appendix to check your answers.

Working Together

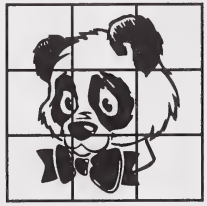
You can make a scale drawing of your favourite cartoon by using tracing paper and a grid.

Example

Make a 200% enlargement of this drawing.

Solution

Step 1: Place a piece of tracing paper over the drawing and then draw a grid on the tracing paper.

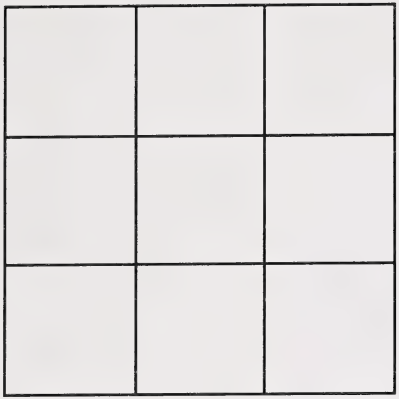


Step 2: The original drawing is 3 cm by 3 cm. Calculate the size of the enlargement.

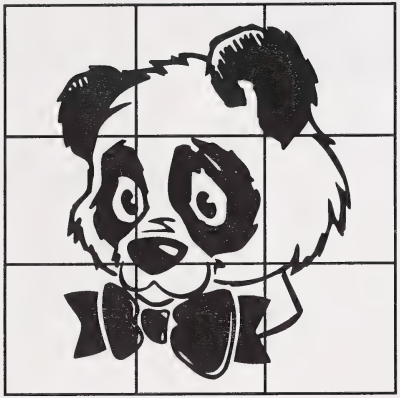
$$200\% \text{ of } 3 = 2 \times 3$$
$$= 6$$

The enlargement will be 6 cm by 6 cm.

Step 3: Make a box that is 6 cm by 6 cm and lighty draw the lines of a grid that corresponds with the original.



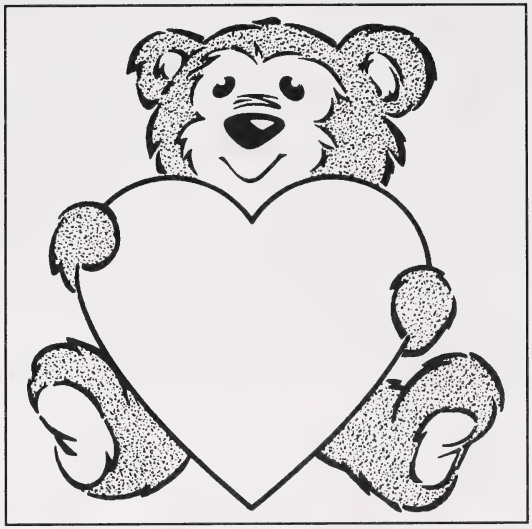
Step 4: Copy the picture from each square on the original grid to the corresponding square on the enlarged grid.



Erase the grid lines.

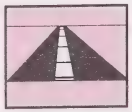
Practice Activity 3

Enlarge the following drawing 200%. Use the method described in this section.



Turn to the Appendix to check your answers.

MODULE CONCLUSION



What Lies Ahead

In the module conclusion you will review the module and do the module assignment.

Working Together

In Module 5 the following skills were taught.

- interpreting ratios
- reading and writing ratios
- expressing ratios in lowest terms
- comparing ratios
- solving ratio problems
- interpreting rates
- reading and writing rates
- simplifying rates
- comparing rates
- solving rate problems
- interpreting percents
- expressing a percent as a decimal number, fraction, whole number, or mixed number
- expressing a fraction, decimal number, whole number, or mixed number as a percent
- solving percent problems
- using the percent key on a calculator
- estimating a percent of a number
- mentally calculating a percent of a number
- calculating simple interest
- solving problems involving interest
- interpreting a scale drawing
- making a scale drawing

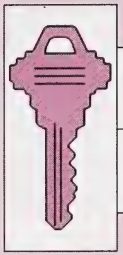
Turn to Section 1 and review the Pretest. Correct any errors you may have made at the time you did the Pretest.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX



Glossary

Suggested Answers

Glossary

Discount: the amount by which a regular price is reduced

Equivalent ratios: ratios that have the same simplest form

$\frac{4}{10}$ and $\frac{6}{15}$ are equivalent ratios.

First term: the first number in a ratio

2 is the first term of $2:3$.

Gear ratio: the ratio of the number of teeth on the driving gear to the number of teeth on the driven gear

Interest: dollars paid for the use of money

Lowest-term ratio: a ratio using the smallest whole numbers possible

Percent: a ratio that compares an amount to 100

85% means $\frac{85}{100}$.

Principal: money invested or borrowed

Proportion: an equation showing that two ratios are equal

$\frac{4}{5} = \frac{24}{30}$ is a proportion.

Proportional: having the same ratio

Proportional ratios: ratios that are equivalent

$\frac{2}{3}$ and $\frac{4}{6}$ are proportional ratios.

Rate: a comparison of quantities with different units

Ratio: a comparison of two number or quantities with the same units

Sales tax: an extra charge added on to the price

Second term: the second number of a ratio

3 is the second term of $2:3$.

Scale: the ratio of the distance between two points on a scale drawing to the distance between the actual points

Scale drawing: a drawing in which all lengths are enlargements or reductions of actual lengths

Term: the numbers in a ratio

$\frac{2}{3}$ ← First term
← Second term

Third term: the third number of a ratio

5 is the third term of $2:3:5$.

Three-term ratios: ratios that have three terms

Turn ratio: the ratio of the number of turns the driving gear makes to the number of turns the driven gear makes

Two-term ratios: ratios that have two terms

Suggested Answers

Section 1: Pretest

See your learning facilitator to check your answers.

Section 2: Practice Activity 1

1. a. The ratio of the amount of oregano to the amount of dill weed is 28 to 13.

b. $13 + 28 = 41$

The ratio of the amount of dill weed to the total amount of spice is 13 to 41.

c. $13 + 28 = 41$

The ratio of the amount of oregano to the total amount of spice is 28 to 41.

2. The ratio of the number of balls to the total number of equipment items is $\frac{7}{10}$.

3. a. The ratio of the number of roses to the number of daisies is 5 : 8.

b. $5 + 8 = 13$

The ratio of the number of roses to the total number of flowers is 5 : 13.

4. $\frac{128}{5} = 25.6$

The student-teacher ratio is 25.6.

5. $\frac{41}{10} = 4.1$

The win-loss ratio is 4.1.

Section 2: Practice Activity 2

1. $\frac{28}{16} \overset{\div 4}{=} \frac{7}{4}$

The ratio of the number of cats to the number of dogs is 7 to 4.

2. $\frac{4}{10} \overset{\div 2}{=} \frac{2}{5}$

The ratio of the temperature in Alberta to the temperature in British Columbia is 2 to 5.

3. a. $\frac{6}{12} \overset{\div 6}{=} \frac{1}{2}$

The gear ratio is $\frac{1}{2}$.

b. Gear A makes two revolutions and Gear B makes one revolution.

c. The turn ratio is $\frac{2}{1}$.

d. The gear ratio and the turn ratio are reciprocals or multiplicative inverses.

4. a.

Gear	Teeth on Front Gear	Teeth on Back Gear	Gear Ratio	Turn Ratio
1st	52	28	$\frac{13}{7}$ or 1.857	$\frac{7}{13}$ or 0.538
2nd	52	24	$\frac{13}{6}$ or 2.167	$\frac{6}{13}$ or 0.462
3rd	52	20	$\frac{13}{5}$ or 2.600	$\frac{5}{13}$ or 0.385
4th	52	17	$\frac{52}{17}$ or 3.059	$\frac{17}{52}$ or 0.327
5th	52	14	$\frac{26}{7}$ or 3.714	$\frac{7}{26}$ or 0.269

b. The gear ratios and the turn ratios are reciprocals or multiplicative inverses.

c. The bicycle is easiest to pedal in first gear. First gear has the least gear ratio and the greatest turn ratio.

d. The bicycle goes fastest in fifth gear. Fifth gear has the greatest gear ratio and the least turn ratio.

5. $\frac{0.47}{4.7} = \frac{47}{470}$ ($\times 100$ on both) and $\frac{47}{470} = \frac{1}{10}$ ($\div 47$ on both)

So, the ratio of the quantity of donated blood to the quantity of blood in the body is 1 to 10.

6. $\frac{0.57}{11.4} = \frac{57}{1140}$ ($\times 100$ on both) and $\frac{57}{1140} = \frac{1}{20}$ ($\div 57$ on both)

The ratio of the volume of the oxygen absorbed to the volume of the air inhaled is 1 to 20.

7. $\frac{0.02}{14.86} = \frac{2}{1486}$ ($\times 100$ on both) and $\frac{2}{1486} = \frac{1}{743}$ ($\div 743$ on both)

The ratio of the length of the Minute Beetle to the length of the Goliath Beetle is 1 to 743.

8. $6\frac{1}{2} : 7\frac{1}{3} = \frac{13}{2} : \frac{22}{3}$

$= \overset{3}{\cancel{6}} \times \frac{13}{\cancel{2}_{1}} : \overset{2}{\cancel{6}} \times \frac{22}{\cancel{3}_{1}}$

$= 39 : 44$

The ratio the time slept on Monday night to the time slept on Tuesday night is 39 to 44.

9. $44\frac{2}{5} : 28\frac{3}{4} = \frac{222}{5} : \frac{115}{4}$

$= \overset{4}{\cancel{20}} \times \frac{222}{\cancel{5}_{1}} : \overset{5}{\cancel{20}} \times \frac{115}{\cancel{4}_{1}}$

$= 888 : 575$

The ratio of the mass of the Hope Diamond to the mass of the Timkin Stone is 888 to 575.

Section 2: Practice Activity 3

1. One photograph is 4 cm by 5 cm. The other phtograph is 8 cm by 10 cm.

   $\frac{5}{4} = \frac{10}{8}$

   Yes, the ratios of the lengths to the widths are equivalent.

2. $\frac{2}{3} \neq \frac{3}{2}$

   No, the ratios of the heights to the diameters are not equivalent.

3. **This year** $\frac{90}{162} \doteq 0.56$ **Last year** $\frac{10}{12} \doteq 0.83$

   $0.83 > 0.56$

   The team is doing better than it did last year.

4. **First game** $\frac{7}{10} = 0.7$ **Second game** $\frac{14}{20} = 0.7$ **Third game** $\frac{3}{5} = 0.6$ **Fourth game** $\frac{3}{4} = 0.75$

   $0.75 > 0.7$ and $0.7 > 0.6$

   a. The player had the greatest success in the fourth game when he made 3 out of 4 shots.

   b. The player had the least success in the third game when he made 3 out of 5 shots.

   c. The player had the same amount of success in the first and second games.

5. **Victor** $\frac{6}{9} \doteq 0.67$ **Aaron** $\frac{9}{15} = 0.6$

   Victor is the more accurate shooter.

6. **First test** $\frac{17}{25} = 0.68$ **Second test** $\frac{14}{20} = 0.7$

   Amelia did better on the second test.

Section 2: Practice Activity 4

1. **a.** The ratio of the quantity of red to yellow to black is $8:5:1$.

   b. The ratio of the quantity of black to red to yellow is $1:8:5$.

   c. The ratio of the quantity of yellow to red to black is $5:8:1$.

   d. $5 + 8 = 14$

   The ratio of the quantity of yellow to brown is $5:14$.

   e. $5 + 8 = 14$

   The ratio of the quantity of black to brown is $1:14$.

   f. $5 + 8 = 14$

   The ratio of the quantity of red to brown is $8:14$ or $4:7$.

2. **a.** The ratio of Albert's age to Mavis' age to Emma's age is $12:13:15$.

b. The ratio of Emma's age to Mavis' age to Albert's age is $15:13:12$.

c. $12+13+15=40$
$12:40=3:10$

The ratio of Albert's age to the total age of all three children is $3:10$.

d. $12+13=25$ and $12+13+15=40$
$25:40=5:8$

The ratio of the sum of Mavis' age and Albert's age to the total age of all three children is $5:8$.

e. $12+13+15=40$
$15:40=3:8$

The ratio of Emma's age to the total age of all three children is $3:8$.

3. $300:450:100=6:9:2$

The ratio of the mass of gravel to sand to cement is $6:9:2$.

4. $36:24:8=9:6:2$

The ratio of wins to losses to ties is $9:6:2$.

5. a. $2:4:2=1:2:1$

The ratio of the number of gold medals to silver medals to bronze medals won by West Germany is $1:2:1$.

b. $5:5:5=1:1:1$

The ratio of the number of gold medals to silver medals to bronze medals won by Switzerland is $1:1:1$.

Section 3: Practice Activity 1

The cross-product method is used in the following suggested answers; however, other methods taught in Module 4, Section 8 may be used.

1. a. $\frac{1}{6}=\frac{2.25}{n}$

$n=13.5$

$\frac{\text{Jump on Earth}}{\text{Jump on moon}}$

A high jumper could jump 13.5 m on the moon.

b. $\frac{1}{6}=\frac{5.5}{n}$

$n=33$

$\frac{\text{Jump on Earth}}{\text{Jump on moon}}$

A pole jumper could jump 33 m on the moon.

2. $\frac{1}{7}\doteq\frac{n}{206}$

$206\doteq 7n$

$29\doteq n$

$\frac{\text{Bones in head}}{\text{Bones in body}}$

There are about 29 bones in your head.

3. $\frac{14}{24}=\frac{n}{52}$

$728=24n$

$30.3\doteq n$

$\frac{\text{Mass of gold}}{\text{Total mass}}$

The mass of the gold in the 14-K bracelet is about 30.3 g.

4. a. $\frac{7}{10} = \frac{n}{120}$

$840 = 10n$

$84 = n$

Pedal revolutions / Wheel revolutions

To make 120 wheel revolutions in first gear, the biker must turn the pedal 84 times.

b. $\frac{1}{3} = \frac{n}{120}$

$120 = 3n$

$40 = n$

Pedal revolutions / Wheel revolutions

To make 120 wheel revolutions in tenth gear, the biker must turn the pedal 40 times.

5. $\frac{2}{5} = \frac{12}{n}$

$2n = 60$

$n = 30$

Passes completed / Passes attemped

Denzil attempted 30 passes.

6. Mass of hydrogen atom / Mass of man

$\frac{1.7 \times 10^{-29}}{70} \doteq \frac{70}{n}$ Mass of man / Mass of sun

$1.7 \times 10^{-29}\, n \doteq 4900$

$1.7 \times 10^{-29} \doteq 4.9 \times 10^{3}$

$n \doteq \frac{4.9}{1.7} \times 10^{3-(-29)}$

$n \doteq 2.88 \times 10^{32}$

The mass of the sun is about 2.88×10^{32} kg.

Note: Question 6 requires a knowledge of operations with numbers expressed in scientific notation. This is taught in Module 3, but is optional content.

Section 3: Practice Activity 2

The cross-product method is used in the following suggested answers; however, other methods taught in Module 4, Section 8 may be used.

1. $4 + 1 = 5$

$\frac{4}{5} \doteq \frac{n}{7.6}$ Small intestine / Total intestines

$30.4 \doteq 5n$

$6.08 \doteq n$

Your small intestine is about 6.08 m long.

2. $37 + 3 = 40$

$\frac{37}{40} = \frac{n}{500}$ Mass of silver / Total mass of alloy

$18\,500 = 40n$

$462.5 = n$

There is about 462.5 g of silver in the goblet.

3. a. $3 + 4 + 2 + 5 = 14$

$\frac{3}{14} = \frac{n}{70}$ Black marbles in Bag A / Total marbles

$210 = 14n$

$15 = n$

There are 15 black marbles in Bag A.

b. $3+4+2+5=14$

$$\frac{4}{14}=\frac{n}{70}$$

$$280=14n$$

$$20=n$$

Red marbles in Bag A / Total marbles

There are 20 red marbles in Bag A.

c. $3+4+2+5=14$

$$\frac{2}{14}=\frac{n}{70}$$

$$140=14n$$

$$10=n$$

Black marbles in Bag B / Total marbles

There are 10 black marbles in Bag B.

d. $3+4+2+5=14$

$$\frac{5}{14}=\frac{n}{70}$$

$$350=14n$$

$$25=n$$

Red marbles in Bag B / Total marbles

There are 25 red marbles in Bag B.

4. Two ways to solve this problem are shown.

Method 1: Using the Guess-Check-Revise Method

The ratio of the original number of girls to the original number of boys was 5 to 7.

So, the original number of girls and the original number of boys was one of these possibilities.

	Original Number of Girls	Original Number of Boys	
$5\times1\rightarrow$	5	7	$\leftarrow 7\times1$
$5\times2\rightarrow$	10	14	$\leftarrow 7\times2$
$5\times3\rightarrow$	15	21	$\leftarrow 7\times3$
$5\times4\rightarrow$	20	28	$\leftarrow 7\times4$
	...	...	

Make guesses about the final number of girls and the final number of boys after 4 girls joined the club. Check to verify that the ratio of the final number of girls to the final number of boys is 1 to 1.

	Final Number of Girls	Final Number of Boys	Test
Guess 1	$5+4=9$	7	$\frac{9}{7}\neq\frac{1}{1}$
Guess 2	$10+4=14$	14	$\frac{14}{14}=\frac{1}{1}$

So, the final number in the club was 14 boys and 14 girls.

Method 2: Using Proportions

The ratio of the original number of girls to the original number of boys was 5 to 7.

So, the original number of girls and the original number of boys was one of these possibilities.

	Original Number of Girls	Original Number of Boys	
$5 \times 1 \rightarrow$	5	7	$\leftarrow 7 \times 1$
$5 \times 2 \rightarrow$	10	14	$\leftarrow 7 \times 2$
$5 \times 3 \rightarrow$	15	21	$\leftarrow 7 \times 3$
$5 \times 4 \rightarrow$	20	28	$\leftarrow 7 \times 4$
	$5n$	$7n$	

If 4 girls joined the club, the final number of girls was $5n + 4$. The final number of boys was $7n$.

$$\frac{5n+4}{7n} = \frac{1}{1}$$

(Girls / Boys)

$$5n + 4 = 7n$$
$$-5n \quad -5n$$
$$4 = 2n$$
$$2 = n$$

Thus, $5n + 4 = 5 \times 2 + 4$ and $7n = 7 \times 2$

$= 10 + 4$ $= 14$

$= 14$

So, the final number in the club was 14 girls and 14 boys.

5. Two ways to solve the problem are shown.

Method 1: Guess-Check-Revise Method

Remember that the profits must be in the ratio of 7 to 5.

So, Sylvia's share and Jane's share will be one of these possibilities.

	Sylvia's Share	Jane's Share	
$7 \times 100 \rightarrow$	700	500	$\leftarrow 5 \times 100$
$7 \times 105 \rightarrow$	735	525	$\leftarrow 5 \times 105$
$7 \times 110 \rightarrow$	770	550	$\leftarrow 5 \times 110$
$7 \times 115 \rightarrow$	805	575	$\leftarrow 5 \times 115$
$7 \times 120 \rightarrow$	840	600	$\leftarrow 5 \times 120$
$7 \times 125 \rightarrow$	875	625	$\leftarrow 5 \times 125$
$7 \times 130 \rightarrow$	910	650	$\leftarrow 5 \times 130$
	...	...	

Make guesses and check. Remember that the total of the two shares must be 1500.

	Sylvia's Share	Jane's Share	Test
Guess 1	805	575	$805 + 575 \neq 1500$
Guess 2	840	600	$840 + 600 \neq 1500$
Guess 3	875	625	$875 + 625 = 1500$

Sylvia's share is $875.

Method 2: Using Proportions

$7 + 5 = 12$

The ratio of Sylvia's share of the profit to the total profit is 7 to 12.

In other words, Sylvia receives \$7 for every \$12 of profit.

$$\frac{7}{12} = \frac{n}{1500}$$

Sylvia's share / Total

$$10\,500 = 12n$$

$$875 = n$$

Sylvia's share is \$875.

Section 3: Practice Activity 3

Proportions and the cross-product method are used in the following suggested answers, but other methods may be used.

1. $5 : 12 : 8 = 100 : b : w$

Red : Blue : White

To find the number of blue marbles, use the first and second terms of each ratio.

$$\frac{5}{100} = \frac{12}{b}$$

Red marbles / Blue marbles

$$5b = 1200$$

$$b = 240$$

There are 240 blue marbles.

To find the number of white marbles, use the first and third term of each ratio.

$$\frac{5}{100} = \frac{8}{w}$$

Red marbles / White marbles

$$5w = 800$$

$$w = 160$$

There are 160 white marbles.

2. $5 + 4 + 1 = 10$

$$\frac{5}{10} = \frac{n}{150\,000}$$

First partner's profit / Total profit

$$750\,000 = 10n$$

$$75\,000 = n$$

The first partner receives \$75 000.

$$\frac{4}{10} = \frac{n}{150\,000}$$

Second partner's profit / Total profit

$$600\,000 = 10n$$

$$60\,000 = n$$

The second partner receives \$60 000.

$$\frac{1}{10} = \frac{n}{150\,000}$$

Third partner's profit / Total profit

$$150\,000 = 10n$$

$$15\,000 = n$$

The third partner receives \$15 000.

3. a. $95 + 4 + 1 = 100$

$$\frac{95}{100} = \frac{n}{4.5}$$

$$427.5 = 100n$$

$$4.3 \doteq n$$

Mass of copper / Total mass

The coin contained about 4.3 g of copper.

b. $98 + 0.5 + 1.5 = 100$

$$\frac{98}{100} = \frac{n}{3.24}$$

$$317.52 = 100n$$

$$3.18 \doteq n$$

Mass of copper / Total mass

The coin contained about 3.18 g of copper.

Section 4: Practice Activity 1

1. a. 2 apples : \$0.75
b. 721 words : 8 min
c. 12.7 cm : 18°C
d. 10.5 cm : 15 g
e. 10^5 messages : 1 s

2. a. $\frac{2 \text{ apples}}{\$0.75}$ **b.** $\frac{721 \text{ words}}{8 \text{ min}}$ **c.** $\frac{12.7 \text{ cm}}{18°C}$

d. $\frac{0.5 \text{ cm}}{15 \text{ g}}$ **e.** $\frac{10^5 \text{ messages}}{1 \text{ s}}$

Section 4: Practice Activity 2

1. a. $\frac{300}{4} \overset{\div 4}{=} \frac{75}{1}$

Beats / Time (min)

Evan's heart rate is 75 beats/min.

b. $\frac{420}{4} \overset{\div 4}{=} \frac{105}{1}$

Words / Time (min)

Bob's typing rate is 105 words/min.

c. $\frac{4}{6} \overset{\div 6}{\doteq} \frac{0.67}{1}$

Stretch (cm) / Mass (g)

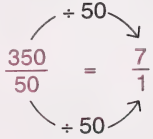
The heavy rubber band stretched about 0.67 cm/g.

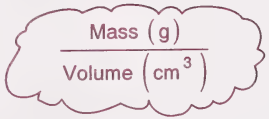
2. a. $\frac{748}{55} \overset{\div 55}{=} \frac{13.6}{1}$

Mass (g) / Volume (cm^3)

The density of mercury is 13.6 g/ cm^3.

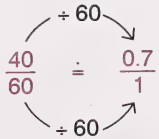
b.

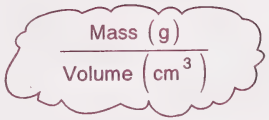




The density of cast iron is 7 g/ cm^3.

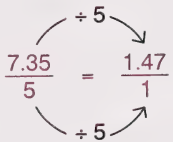
c.

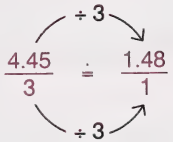




The density of oak is about 0.7 g/ cm^3.

3. **First price** **Second price**

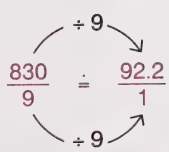




$7.35/5 kg < $4.45/3 kg

The better buy is 5 kg of bananas for $7.35.

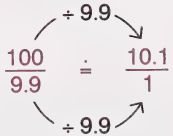
4. **Dino** **Frank**

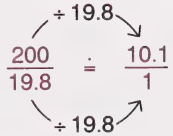


92.5 km/h > 91.4 km/h

Dino is faster than Frank.

5. **First athlete** **Second athlete**



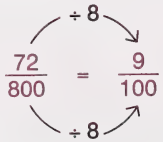


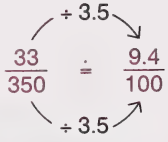
100 m/9.9 s = 200 m/19.8 s

The athletes are running at the same rate.

Section 4: Practice Activity 3

1. **Miguel** **Rico**



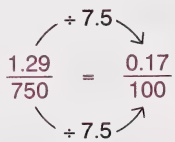


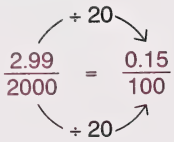
72 L/800 km < 33 L/350 km

Miguel's car has the better rate of gas consumption.

2. **Price of 750-mL bottle** **Price of 2-L bottle**

2 L = 2000 mL

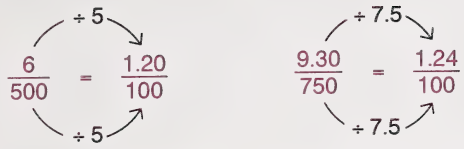




$1.29/750 mL > $2.99/2L

The better buy is a 2-L bottle for $2.99.

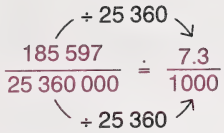
3. **Ruth** **Yvonne**



$6/500 g < $9.30/750 g

Ruth received the better buy.

4.



Marriages / Population

In Canada in 1985 there were 7.3 marriages/1000 population.

Section 5: Practice Activity 1

Proportions and the cross-product methods are shown but other methods may be used.

1. a. $\frac{6}{30} = \frac{n}{1}$

$6 = 30n$

$0.2 = n$

Distance (km) / Time (min)

Arthur's training rate of speed is 0.2 km/min.

b. $\frac{6}{30} = \frac{10}{n}$

$6n = 300$

$n = 50$

Distance (km) / Time (min)

At this rate, it will take Arthur 50 min to jog 10 km.

c. $\frac{6}{30} = \frac{n}{45}$

$270 = 30n$

$9 = n$

Distance (km) / Time (min)

Arthur could jog 9 km in 45 min.

2.

Kilograms of meat	2	4	10
Cost in dollars	8.60	17.20	43.00

3. $\frac{8}{10.99} = \frac{10}{n}$

$8n = 109.9$

$n = 13.74$

Socks / Cost

The price of 10 pairs of socks would be $13.74.

4. $\frac{72}{1} = \frac{n}{30}$

$2160 = n$

Beats / Time (min)

In 30 min, the heart beats about 2160 times.

5. DC-9 Plane

$\frac{826}{1} = \frac{n}{3}$

$2478 = n$

Distance (km) / Time (h)

The DC-9 plane can travel 2478 km in 3 h.

747 Plane

$\frac{893}{1} = \frac{n}{3}$

$2679 = n$

Distance (km) / Time (h)

The 747 plane can travel 2679 km in 3 h.

L-1011 Plane

$\frac{882}{1} = \frac{n}{3}$

$2646 = n$

Distance (km) / Time (h)

The L-1011 plane can travel 2646 km in 3 h.

6. $\frac{450}{40} = \frac{n}{36}$

$16\,200 = 40n$

$405 = n$

Earnings ($) / Time (h)

René will receive $405 if she works a 36-h week.

7. $\frac{28.4}{2.25} = \frac{50}{n}$

$28.4n = 112.5$

$n \doteq 4$

Distance (km) / Time (h)

The bicyclist will travel 50 km in about 4 h.

8. $\frac{2.5}{1} = \frac{n}{50}$

$125 = n$

Distance (m) / Time (s)

Arlene can swim 125 m at this rate.

9. $\frac{17}{100} = \frac{n}{2500}$

$42\,500 = 100n$

$425 = n$

Consumption (L) / Distance (km)

So, 425 L of gasoline would be needed.

10. You may be tempted to set up a proportion like this.

$\frac{1}{3} = \frac{4}{n}$

$n = 12$

Eggs / Time (min)

However, if you think about the problem more carefully, you will realize you can boil 4 eggs at once.

So, it takes only 4 min to boil 4 eggs.

Section 5: Practice Activity 2

1. $1\text{ h} = 60\text{ min}$

$\frac{25}{1} = \frac{n}{60}$

$1500 = n$

$\frac{\text{Blinks}}{\text{Time (min)}}$

Your eye blinks about 1500 times in an hour.

2. $1\text{ h} = 60\text{ min}$

$24\text{ h} = 1\text{ d}$

$1\text{ d} = 1440\text{ min}$

$\frac{80}{1} = \frac{n}{1440}$

$115\,200 = n$

$\frac{\text{Beats}}{\text{Time (min)}}$

Your heart beats about 115 200 times in a day.

3. $5\text{ km} = 5000\text{ m}$

$\frac{5}{1} = \frac{5000}{m}$

$5m = 5000$

$m = 1000$

$\frac{\text{Distance (m)}}{\text{Time (min)}}$

It will take 1000 min.

$1\text{ h} = 60\text{ min}$

$1000\text{ min} \doteq 16.7\text{ h}$

It will take about 16.7 h for the Mauritius tortoise to travel 5 km.

4. $\frac{5}{1} = \frac{100}{n}$

$5n = 100$

$n = 20$

$\frac{\text{Distance (m)}}{\text{Time (s)}}$

It would take Gino 20 s to run the 100-m race.

$20 - 5 = 15$

If Gino had a head start of 5 s, it would take him 15 s to run the 100-m race.

$\frac{7}{1} = \frac{100}{n}$

$7n = 100$

$n = 14.3$

$\frac{\text{Distance (m)}}{\text{Time (s)}}$

It would take the athlete 14.3 s to run the 100-m race.

$14.3\text{ s} < 15\text{ s}$

No, Gino would not win the race even if he had a 5-s head start.

5. $\frac{6}{100} = \frac{60}{n}$

$6n = 60\,000$

$n = 1000$

$\frac{\text{Consumption (L)}}{\text{Distance (km)}}$

Raja's car can travel 1000 km on 60 L of gas.

$$\frac{15}{100} = \frac{n}{1000}$$

$$15\,000 = 100n$$

$$150 = n$$

Consumption (L) / Distance (km)

Helmut's van consumes 150 L to travel as far as Raja's car travels on 60 L.

6. $$\frac{51.2}{100} = \frac{n}{200}$$

$$10\,240 = 100n$$

$$102.4 = n$$

Distance (m) / Time (s)

She would swim 200 m in 102.4 s.

1 min = 60 s

102.4 – 60 = 42.4

So, 102.4 s = 1 min 42.4 s.

$$\begin{array}{r} 1\text{ min }52.8\text{ s} \\ -1\text{ min }42.4\text{ s} \\ \hline 10.4\text{ s} \end{array}$$

If the swimmer could swim at this rate, she would cut 10.4 s from her record.

7. $$\frac{8.60}{1} = \frac{n}{40}$$

$$344 = n$$

Earnings ($) / Time (h)

Larissa earns $344 for a 40-h week.

50 – 40 = 10

Larissa worked 10 h of overtime.

$$\frac{14.75}{1} = \frac{n}{10}$$

$$n = 147.50$$

Earnings ($) / Time (h)

Larissa earns $147.50 for 10 h of overtime.

$$\begin{array}{r} 344.00 \\ +\ 147.50 \\ \hline 491.50 \end{array}$$

Larissa earns $491.50 for a 50-h week.

Section 5: Practice Activity 3

1. Susan can paint 1 room in 3 h.

Susan can paint $\frac{1}{3}$ of a room in 1 h.

Sherry can paint 1 room in 4 h.

Sherry can paint $\frac{1}{4}$ of a room in 1 h.

Calculate the rate if they work together.

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12}$$

$$= \frac{7}{12}$$

Susan and Sherry working together can paint $\frac{7}{12}$ of the room in 1 h.

Calculate the number of hours it will take to paint the whole room.

$$\frac{\frac{7}{12}}{1} = \frac{1}{n}$$

$$\frac{7}{12}n = 1$$

$$n = \frac{12}{7}$$

$$\doteq 1.7$$

Rooms / Time (h)

It will take about 1.7 h.

2. Tap A fills 1 sink in 8 min.

Tap A fills $\frac{1}{8}$ of the sink in 1 min.

Tap B fills 1 sink in 6 min.

Tap B fills $\frac{1}{6}$ of the sink in 1 min.

Calculate the rate per minute if both taps are working.

$$\frac{1}{8} + \frac{1}{6} = \frac{3}{24} + \frac{4}{24}$$

$$= \frac{7}{24}$$

Tap A and Tap B will fill $\frac{7}{24}$ of the sink in 1 min.

Calculate how long it will take to fill the whole sink if both taps are working together at this rate.

$$\frac{\frac{7}{24}}{1} = \frac{1}{n}$$

$$\frac{7}{24}n = 1$$

$$n = \frac{24}{7}$$

$$\doteq 3.4$$

Sinks / Time (min)

It will take about 3.4 min.

3. Todd and Tim working together can mow 1 park lawn in 2 h.

Todd and Tim working together can mow $\frac{1}{2}$ of the park lawn in 1 h.

Todd working alone can mow 1 park lawn in 3 h.

Todd working alone can mow $\frac{1}{3}$ of the park lawn in 1 h.

Calculate Tim's hourly rate of mowing.

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6}$$

$$= \frac{1}{6}$$

Tim working alone can mow $\frac{1}{6}$ of the park lawn in 1 h.

Write a proportion to discover how long it will take Tim to mow the whole lawn alone.

$$\frac{\frac{1}{6}}{1} = \frac{1}{n}$$

$$n = 6$$

Lawn / Time (h)

It will take Tim 6 h to mow the whole park lawn alone.

4. Altogether, 6 hens can lay 24 eggs in 6 d.
Altogether, 6 hens can lay 4 eggs in 1 d.
Altogether, 3 hens can lay 2 eggs in 1 d.

Calculate how many eggs 3 hens can lay in 3 d.

$$\frac{2}{1} = \frac{n}{3}$$

$$6 = n$$

Eggs / Time (d)

Altogether, 3 hens can lay 6 eggs in 3 d.

Section 6: Practice Activity 1

1. $d = vt$
$= 80 \times 3$
$= 240$

Martin travelled 240 km.

2. $d = vt$
$= 4.2 \times 45$
$= 189$

The balloon rose 189 m in 45 s.

3. $d = vt$
$2 = v \times 45$
$\frac{2}{45} = v$

Troy walks at a speed of 2 km/45 min.

$$\frac{2}{45} \xrightarrow{\div 45} \frac{0.04}{1}$$

Distance (km) / Time (min)

Troy walks at a rate of about 0.04 km/min.

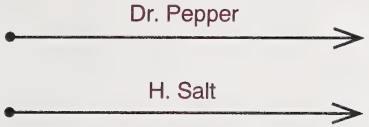
4. $d = vt$
$45 = 3.5 \times t$
$12.9 \doteq t$

The elevator will take about 12.9 s to descend 45 m.

Section 6: Practice Activity 2

The problems can be solved using the guess-check-revise method or using equations. Only the equation method is shown.

1.



9 a.m. to 4 p.m. is 7 h.
11 a.m. to 4 p.m. is 5 h.

Calculate the distance H. Salt travelled. It is the same distance that Dr. Pepper travelled.

$d = vt$
$= 60 \times 7$
$= 420$

H. Salt travelled 420 km.

Calculate H. Salt's speed.

$d = vt$
$420 = v \times 5$
$84 = v$

H. Salt drove at a speed of 84 km/h.

2.

Let the time be t.

Organize the data in a table and calculate the distanced travelled by each truck using the formula $d = vt$.

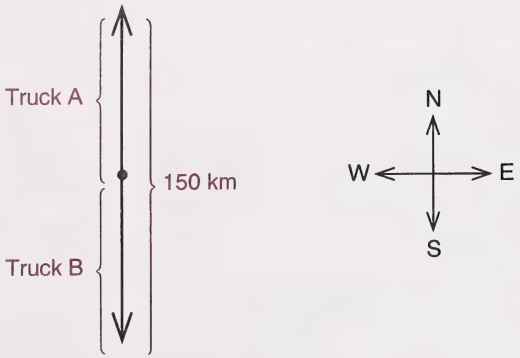
	d	v	t
Truck A	$70t$	70	t
Truck B	$80t$	80	t

Together the trucks travelled 900 km. Write an equation to show this.

$70t + 80t = 900$
$150t = 900$
$t = 6$

The trucks were 900 km apart after 6 h.

3.



Let the time Truck A was travelling be t.
Let the time Truck B was travelling be $t-1$.

Organize the data in a table and calculate the distance travelled by each truck using the fomula $d = vt$.

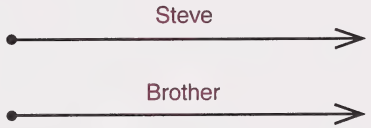
	d	v	t
Truck A	$80t$	80	t
Truck B	$60t - 60$	60	$t-1$

Together the trucks travelled 150 km. Write an equation to show this.

$$\begin{array}{rl} 140t - \cancel{60} = & 150 \\ +\cancel{60} & +60 \\ \hline 140t = & 210 \\ t = & 1.5 \end{array}$$

The first truck had travelled 1.5 h when they were 150 km apart.

4. a.



8:00 a.m. to 10:00 a.m. is 2 h. So, Steve's brother left 2 h after Steve.

Let the time that Steve travelled be t.
Let the time that the brother travelled be $t-2$.

Organize the data in a table and calculate the distance travelled by each person using the formula $d = vt$.

	d	v	t
Steve	$18t$	18	t
Brother	$54t - 108$	54	$t-2$

The distance each person travelled is the same. Write an equation to show this.

$$\begin{array}{rl} 18t = & \cancel{54t} - 108 \\ -54t & -\cancel{54t} \\ \hline -36t = & -108 \\ 36t = & 108 \\ t = & 3 \end{array}$$

Steve had travelled 3 h when his brother overtook him.

b. If $t = 3$ then,

$$18t = 18 \times 3$$
$$= 54$$

Steve had travelled 54 km.

Section 7: Practice Activity 1

1. a. $60\% = \frac{60}{100}$
$= \frac{3}{5}$

b. $60\% = \frac{60}{100}$
$= 0.6$

2. a. $45\% = \frac{45}{100}$
$= \frac{9}{20}$

b. $45\% = \frac{45}{100}$
$= 0.45$

3. a. $60\% = \frac{60}{100}$
$= \frac{3}{5}$

b. $60\% = \frac{60}{100}$
$= 0.60$
$= 0.6$

4. a. $87\frac{1}{2}\% = 87.5\%$
$= \frac{87.5}{100}$
$= \frac{875}{1000}$
$= \frac{7}{8}$

b. $87\frac{1}{2}\% = 87.5\%$
$= \frac{87.5}{100}$
$= \frac{875}{1000}$
$= 0.875$

5. a. $66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100}$
$= \frac{\frac{200}{3}}{100}$
$= \frac{200}{3} \div 100$
$= \frac{200}{3} \times \frac{1}{100}$
$= \frac{2}{3}$

b. $66\frac{2}{3}\% \doteq 66.7\%$
$\doteq \frac{66.7}{100}$
$\doteq \frac{667}{1000}$
$\doteq 0.667$

6. a. $2.5\% = \frac{2.5}{100}$
$= \frac{25}{1000}$
$= \frac{1}{40}$

b. $2.5\% = \frac{2.5}{100}$
$= \frac{25}{1000}$
$= 0.025$

7. a. $\frac{1}{30}\% = \frac{\frac{1}{30}}{100}$
$= \frac{1}{30} \div 100$
$= \frac{1}{30} \times \frac{1}{100}$
$= \frac{1}{3000}$

b. $\frac{1}{30}\% \doteq 0.03\%$
$\doteq \frac{0.03}{100}$
$\doteq \frac{3}{10\,000}$
$\doteq 0.0003$

Section 7: Practice Activity 2

1. $300\% = \frac{300}{100}$
$= 3$

2. a. $110\% = \frac{110}{100}$
$= 1\frac{10}{100}$
$= 1\frac{1}{10}$

b. $100\% = \frac{110}{100}$
$= 1.10$
$= 1.1$

3. a. $125\% = \frac{125}{100}$
$= 1\frac{25}{100}$
$= 1\frac{1}{4}$

b. $125\% = \frac{125}{100}$
$= 1.25$

4. $167\,000\% = \frac{167\,000}{100}$
$= 1670$

5. $5000\% = \frac{5000}{100}$
$= 50$

Section 7: Practice Activity 3

1. a. $\frac{3}{4} = 3 \div 4$
$= 0.75$

b. $\frac{3}{4} = \frac{75}{100}$
$= 75\%$

2. a. $\frac{1}{6} = 1 \div 6$
$\doteq 0.17$

b. $\frac{1}{6} = \frac{n}{100}$
$100 = 6n$
$\frac{100}{6} = n$
$16\frac{2}{3}\% = n$

3. a. $0.25 = \frac{25}{100}$
$= \frac{1}{4}$

b. $0.25 = \frac{25}{100}$
$= 25\%$

4. a. $0.3 = \frac{3}{10}$

b. $0.3 = \frac{3}{10}$
$= \frac{30}{100}$
$= 30\%$

Section 7: Practice Activity 4

1. $2.2 = \frac{22}{10}$
$= \frac{220}{100}$
$= 220\%$

2. $5\frac{3}{4} = \frac{23}{4}$
$= \frac{575}{100}$
$= 575\%$

3. $2\frac{1}{3} = \frac{7}{3}$

$= \frac{700}{300}$

$= \frac{233\frac{1}{3}}{100}$

$= 233\frac{1}{3}\%$

4. $1\frac{2}{3} = \frac{5}{3}$

$= \frac{500}{300}$

$= \frac{166\frac{2}{3}}{100}$

$= 166\frac{2}{3}\%$

5. $1000\,000 = \frac{1000\,000}{100}$

$= 100\,000\,000\%$

Section 7: Practice Activity 5

1. a. Think $\frac{1}{10} = 10\%$

$\frac{7}{10} = 7 \times \frac{1}{10}$

$= 7 \times 10\%$

$= 70\%$

b. Think $\frac{1}{100} = 1\%$

$\frac{9}{100} = 9 \times \frac{1}{100}$

$= 9 \times 1\%$

$= 9\%$

c. Think $\frac{1}{10} = 10\%$

$\frac{1}{20} = \frac{1}{2} \times \frac{1}{10}$

$= \frac{1}{2} \times 10\%$

$= 5\%$

d. Think $\frac{1}{100} = 1\%$

$\frac{1}{200} = \frac{1}{2} \times \frac{1}{100}$

$= \frac{1}{2} \times 1\%$

$= 0.5\%$

e. Think $\frac{1}{4} = 25\%$

$\frac{1}{16} = \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{4} \times 25\%$

$= 6\frac{1}{4}\%$

f. Think $\frac{1}{10} = 10\%$

$\frac{1}{5} = 2 \times \frac{1}{10}$

$= 2 \times 10\%$

$= 20\%$

2. a. Think $\frac{1}{10} = 10\%$

$\frac{1}{20} = \frac{1}{2} \times 10\%$

$= 5\%$

$\frac{3}{20} = 3 \times \frac{1}{20}$

$= 3 \times 5\%$

$= 15\%$

b. Think $\frac{1}{100} = 1\%$

$\frac{1}{200} = \frac{1}{2} \times \frac{1}{100}$

$= \frac{1}{2} \times 1\%$

$= 0.5\%$

$\frac{7}{200} = 7 \times \frac{1}{200}$

$= 7 \times 0.5\%$

$= 3.5\%$

c. Think $\frac{1}{4} = 25\%$

$$\frac{1}{8} = \frac{1}{2} \times \frac{1}{4}$$
$$= \frac{1}{2} \times 25\%$$
$$= 12\frac{1}{2}\%$$

$$\frac{5}{8} = 5 \times \frac{1}{8}$$
$$= 5 \times 12\frac{1}{2}\%$$
$$= 62\frac{1}{2}\%$$

d. Think $\frac{1}{10} = 10\%$

$$\frac{1}{5} = 2 \times \frac{1}{10}$$
$$= 2 \times 10\%$$
$$= 20\%$$

$$\frac{4}{5} = 4 \times \frac{1}{5}$$
$$= 4 \times 20\%$$
$$= 80\%$$

Section 8: Practive Activity 1

The proportion method is shown, but students may write an equation instead.

1. $\frac{n}{5200} = \frac{140}{100}$

$100n = 728\,000$

$n = 7280$

Profits ($) / Sales ($)

The profits this year were $7280.

2. $\frac{10}{16} = \frac{n}{100}$

$16n = 1000$

$n = 62.5$

Klutz got 62.5% correct.

3. $\frac{n}{750} = \frac{0.2}{100}$

$100n = 150$

$n = 1.5$

Increase (m) / Length (m)

The cable will be 1.5 m longer.

4. $13 + 15 + 2 = 30$

$\frac{13}{30} = \frac{n}{100}$

$1300 = 30n$

$43.3 \doteq n$

Wins / Total games played

The team won about 43.3% of the games.

5. $\frac{5000}{n} = \frac{40}{100}$

$500\,000 = 40n$

$12\,500 = n$

Earnings / Sales

$12 500 worth of magazines must be sold.

6. $\frac{1000}{n} = \frac{2.5}{100}$

$100\,000 = 2.5n$

$40\,000 = n$

Commission / Sales

The broker needs $40 000 in sales to earn a commission of $1000.

7. $\frac{54}{80} = \frac{n}{100}$

$5400 = 80n$

$67.5 = n$

Baskets / Free throws

Elmo makes 67.5% of the free throws.

$100\% - 67.5\% = 32.5\%$

Elmo misses 32.5% of the free throws.

Section 8: Practice Activity 2

1. **a.** $25\% \text{ of } 950 = 0.25 \times 950$

$= 237.50$

$950.00 - 237.50 = 712.50$

The sale price is $712.50

b. $20\% \text{ of } 249.99 = 0.2 \times 249.99$

$\doteq 50.00$

$249.99 - 50.00 = 199.99$

The sale price is $199.99.

2. **a.** British Columbia calculates the GST and PST side by side.

$7\% \text{ of } 89.99 \doteq 0.07 \times 89.99$

$\doteq 6.30$

$6\% \text{ of } 89.99 \doteq 0.06 \times 89.99$

$\doteq 5.40$

$89.99 + 6.30 + 5.40 = 101.69$

The cost in British Columbia is $101.69.

b. Alberta only has GST added on to the price.

$7\% \text{ of } 17.50 = 0.07 \times 17.50$

$\doteq 1.23$

$17.50 + 1.23 = 18.73$

The cost in Alberta is $18.73.

c. Nova Scotia calculates the PST after the GST has been added on to the price.

$7\% \text{ of } 49.95 = 0.07 \times 49.95$

$\doteq 3.50$

$49.95 + 3.50 = 53.45$

$10\% \text{ of } 53.45 = 0.1 \times 53.45$

$\doteq 5.35$

$53.45 + 5.35 = 58.80$

The cost in Nova Scotia is $58.80.

Section 8: Practice Activity 3

1. **a.** $100\% - 30\% = 70\%$

$70\% \text{ of } 35 = 0.7 \times 35$

$= 24.50$

The sale price is $24.50.

b. $100\% - 30\% = 70\%$

$70\% \text{ of } 89.99 = 0.7 \times 89.99$

$\doteq 63.00$

The sale price is $63.00.

2. $100\% + 7\% = 107\%$

107% of $18\,500 = 1.07 \times 18\,500$

$= 19\,795$

The cost is $19 795.

3. In Manitoba GST and PST are calculated side by side.

$100\% + 7\% + 7\% = 114\%$

114% of $39 = 1.14 \times 39$

$= 44.46$

The cost is $44.46.

Section 8: Practice Activity 4

1. There are several ways to solve this problem. Three methods are shown.

Method 1: Guess-Check-Revise

Number of Cattle	First Son's Share	Second Son's Share	Third Son's Share	Fourth Son's Share	Test
60	30	15	12	7	$64 \neq 60$
120	60	30	24	7	$121 \neq 120$
140	70	35	28	7	$140 = 140$

The farmer had 140 cattle.

Method 2: Using an Equation

Let n be the number of cattle.
Let $0.5n$ be the first son's share.
Let $0.25n$ be the second son's share.
Let $0.2n$ be the third son's share.
The fourth son's share is 7.

$$0.5n + 0.25n + 0.2n + 7 = n$$
$$0.95n + 7 = n$$
$$-0.95n \quad -0.95n$$
$$7 = 0.05n$$
$$\frac{7}{0.05} = \frac{0.05n}{0.05}$$
$$140 = n$$

The farmer had 140 cattle.

Method 3

$50\% + 25\% + 20\% = 95\%$

$100\% - 95\% = 5\%$

5% of _____ $= 7$

$0.05n = 7$

$$\frac{0.05n}{0.05} = \frac{7}{0.05}$$

$n = 140$

The farmer had 140 cattle

2. a. Method 1

Let n be the regular price
Let $0.25n$ be the discount.

$0.25n = 37.50$

$$\frac{\cancel{0.25}^{1}n}{\cancel{0.25}_{1}} = \frac{37.50}{0.25}$$

$n = 150$

The regular price is $150.

Method 2

$$\frac{25}{100} = \frac{37.50}{n}$$

$n = 150$

Discount / Regular price

The regular price is $150.

b. Method 1

$150 - 37.50 = 112.50$

The sale price is $112.50.

Method 2

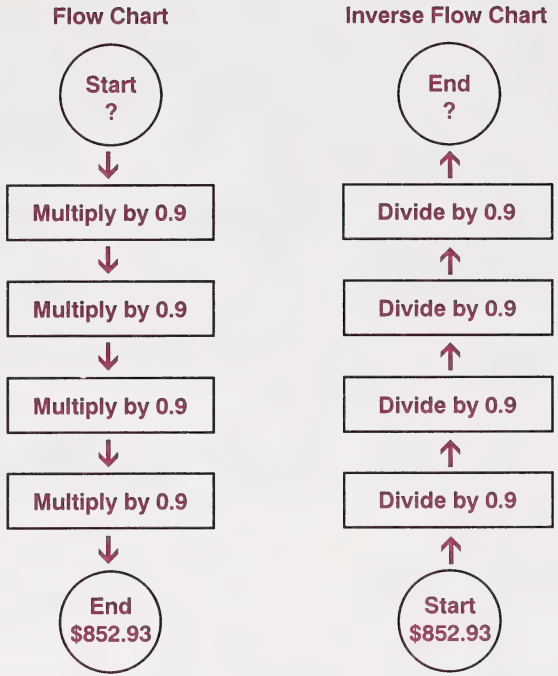
Reducing the jacket by 25% is the same as selling it for 75% of the regular price.

$75\% \text{ of } 150 = 0.75 \times 150$
$= 112.50$

The sale price is $112.50.

3. Work backwards.

Reducing the stereo by 10% is the same as selling it for 90% of the regular price.



$852.93 \div 0.9 \div 0.9 \div 0.9 \div 0.9 = 1300$

The original price was $1300.

Section 9: Practice Activity 1

Print Alternative

1. A range of answers is acceptable for these questions. Only one method is given for each question, but several are possible.

   a. Think 98% of 680 $\doteq$ 100% of 680
   $\doteq$ 680

   b. Think 26% of 399 $\doteq$ 30% of 400
   $\doteq 0.3 \times 400$
   $\doteq 120$

   c. Think 32% of 180 $\doteq 33\frac{1}{3}\%$ of 180
   $\doteq \frac{1}{3}$ of 180
   $\doteq 60$

   d. Think 49% of 105 $\doteq$ 50% of 105
   $\doteq \frac{1}{2}$ of 105
   $\doteq 52.5$

   e. Think 11% of 58 $\doteq$ 10% of 58
   $\doteq 0.1 \times 58$
   $\doteq 5.80$

Computer Alternative

2. computer checked

Section 9: Practice Activity 2

1. **a.** Think 10% of 48 = 4.8

   b. Think 10% of 7.25 = 0.725

   c. Think 1% of 73 = 0.73

   d. Think 1% of 5.23 = 0.0523

   e. Think 10% of 35 = 3.50
   20% of 35 = 7

   f. Think 10% of 9.20 = 0.92
   20% of 9.20 = 1.84

   g. Think 50% of 46 $= \frac{1}{2}$ of 46
   = 23

   h. Think 50% of 8.20 $= \frac{1}{2}$ of 8.20
   = 4.10

   i. Think $33\frac{1}{3}\%$ of 123 $= \frac{1}{3}$ of 123
   = 41

   j. Think $33\frac{1}{3}\%$ of 4.50 $= \frac{1}{3}$ of 4.50
   = 1.50

   k. Think $33\frac{1}{3}\%$ of 30 $= \frac{1}{3}$ of 30
   = 10
   $66\frac{2}{3}\%$ of 30 = 20

l. Think $33\frac{1}{3}\%$ of $3.60 = \frac{1}{3}$ of 3.60

$= 1.20$

$66\frac{2}{3}\%$ of $3.60 = 2.40$

m. Think 25% of $88 = \frac{1}{4}$ of 88

$= 22$

n. Think 25% of $12.80 = \frac{1}{4}$ of 12.80

$= 3.20$

o. Think 25% of $80 = \frac{1}{4}$ of 80

$= 20$

75% of $80 = 60$

p. Think 25% of $16.80 = \frac{1}{4} \times 16.80$

$= 4.20$

$12\frac{1}{2}\%$ of $16.88 = 2.10$

2. **a.** 9 **b.** 9 **c.** 22 **d.** 22

3. 18% of $50 = 50\%$ of 18
88% of $25 = 25\%$ of 88

So, $a\%$ of $b = b\%$ of a.

4. **a.** Think 26% of $50 = 50\%$ of 26

$= \frac{1}{2}$ of 26

$= 13$

b. Think 84% of $25 = 25\%$ of 84

$= \frac{1}{4}$ of 84

$= 21$

c. Think 55% of $20 = 20\%$ of 55

$= \frac{1}{5}$ of 55

$= 11$

5. **a.** 27 **b.** 27 **c.** 25.2 **d.** 25.2
e. 4.56 **f.** 4.56

6. 45% of $60 = 20\%$ of $60 + 25\%$ of 60
60% of $42 = 50\%$ of $42 + 10\%$ of 42
19% of $24 = 20\%$ of $24 - 1\%$ of 24

7. **a.** Think 35% of $80 = 25\%$ of $80 + 10\%$ of 80

$= \frac{1}{4}$ of $80 + 8$

$= 20 + 8$

$= 28$

b. Think 31% of $60 = 30\%$ of $60 + 1\%$ of 60

$= 0.3 \times 60 + 0.6$

$= 18 + 0.6$

$= 18.6$

c. Think 79% of 50 = 80% of 50 − 1% of 50

$= 0.8 \times 50 - 0.5$

$= 40 - 0.5$

$= 39.5$

d. Think 11% of 40 = 10% of 40 + 1% of 40

$= 4 + 0.4$

$= 4.4$

8. **a.** 12 **b.** 12 **c.** 10.5 **d.** 10.5
e. 45 **f.** 45

9. 25% of 48 = 50% of 24
15% of 70 = 30% of 35
90% of 50 = 45% of 100

a% of $b = 2a$% of $\frac{b}{2}$, and a% of $b = \frac{a\%}{2}$ of $2b$.

10. **a.** Think 15% of 32 = 30% of 16

$= 0.3 \times 16$

$= 4.8$

b. Think 50% of 36 = 100% of 18

$= 18$

c. 20% of 50 = 100% of 10

$= 10$

d. 36% of 200 = 9% of 800

$= 0.09 \times 800$

$= 72$

e. 24% of 20 = 3% of 160

$= 0.03 \times 160$

$= 4.8$

f. 35% of 60 = 70% of 30

$= 0.7 \times 30$

$= 21$

Section 10: Practice Activity 1

1. $I = Prt$

$= 1500 \times 0.06 \times 3$

$= 270$

$270 in interest is earned.

2. $I = Prt$

$= 12\,000 \times 0.125 \times 2$

$= 3000$

$3000 in interest is earned.

3. $I = Prt$

$= 7500 \times 0.08 \times 1.5$

$= 900$

Interest of $900 is paid.

Section 10: Practice Activity 2

1. $I = Prt$

$= 3000 \times 0.05 \times \frac{3}{12}$

$= 37.50$

Interest of $37.50 is paid.

2. $I = Prt$

$= 6500 \times 0.115 \times \frac{90}{365}$

$= 184.32$

Interest of $184.32 is made.

3. $1\frac{1}{4}\%/\text{mo} = 15\%/\text{a}$

$I = Prt$

$= 450 \times 0.15 \times 2$

$= 135$

Interest of \$135 is paid.

4. $2\%/\text{mo} = 24\%/\text{a}$

$I = Prt$

$= 3900 \times 0.24 \times \frac{5}{12}$

$= 390$

Interest of \$390 is paid.

Section 10: Practice Activity 3

Two different methods are shown for the following problems.

1. Method 1: Guess-Check-Revise Method

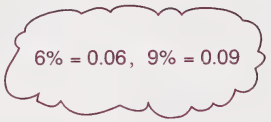
Notice the amount invested at 9% is twice the amount invested at 6%.

	Savings Invested at 6%	Savings Invested at 9%	Test
Guess 1	100	200	$6 + 18 \neq 72$
Guess 2	200	400	$12 + 36 \neq 72$
Guess 3	300	600	$18 + 54 = 72$

Ms. Twinkle invested \$300 at 6% and \$600 at 9%.

Method 2: Writing an Equation

Let the savings invested at 6% be x.
Let the savings invested at 9% be $2x$.



$0.06x + 0.09(2x) = 72$

$0.06x + 0.18x = 72$

$0.24x = 72$

$x = 300$

Ms. Twinkle invested \$300 at 6%.

If $x = 300$, then

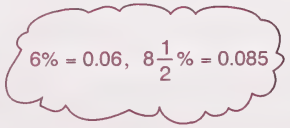
$2x = 2 \times 300$

$= 600$

Ms. Twinkle invested \$600 at 9%.

2. **Method 1: Guess-Check-Revise Method**

The scholarship fund totals \$7000.
So the sum of two parts must be \$7000.

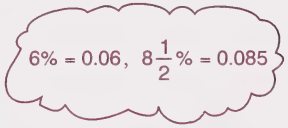


	Part Invested at 6%	Part Invested at $8\frac{1}{2}\%$	Test
Guess 1	1000	6000	$60 + 510 \neq 520$
Guess 2	2000	5000	$120 + 425 \neq 520$
Guess 3	3000	4000	$180 + 340 = 520$

There was \$3000 invested at 6% and \$4000 invested at $8\frac{1}{2}\%$.

Method 2: Writing an Equation

Let the part invested at 6% be x.
Let the part invested at $8\frac{1}{2}\%$ be $7000 - x$.



$$0.06x + 0.085(7000 - x) = 520$$
$$0.06x + 595 - 0.085x = 520$$
$$-0.025x + 595 = 520$$
$$-0.025x = -75$$
$$x = 3000$$

There was \$3000 invested at 6%.

If $x = 3000$, then

$$7000 - x = 7000 - 3000$$
$$= 4000$$

There was \$4000 invested at $8\frac{1}{2}\%$.

3. Method 1: Guess-Check-Revise Method

The part invested at 7% was $200 more than the part invested at 13%.

	Part Invested at 13%	Part Invested at 7%	Test
Guess 1	300	500	$39 + 35 \neq 84$
Guess 2	400	600	$52 + 42 \neq 84$
Guess 3	350	550	$45.50 + 38.50 = 84$

Method 2: Writing an Equation

Let the amount invested at 13% be x.
Let the amount invested at 7% be $x + 200$.

13% = 0.13, 7% = 0.07

$$0.13x + 0.07(x + 200) = 84$$
$$0.13x + 0.07x + 14 = 84$$
$$0.2x + 14 = 84$$
$$0.2x = 70$$
$$x = 350$$

There was $350 invested at 13%.

If $x = 350$, then

$$x + 200 = 350 + 200$$
$$= 550$$

There was $550 invested at 7%.

Section 11: Practice Activity 1

1. Scale: 1 cm represents 15 m.

a. The dwelling in the scale drawing is 2 cm by 0.6 cm.

Calculate the actual length.

$$\frac{2}{\ell} = \frac{1}{15}$$
$$30 = \ell$$

Drawing (cm) / Actual (m)

The actual dwelling was 30 m long.

Calculate the actual width.

$$\frac{0.6}{w} = \frac{1}{15}$$
$$9 = w$$

Drawing (cm) / Actual (m)

The actual dwelling was 9 m wide.

b. The office in the diagram is 1.0 cm by 0.3 cm.

Calculate the actual length.

$$\frac{1.0}{\ell} = \frac{1}{15}$$

$$15 = \ell$$

$\frac{\text{Drawing (cm)}}{\text{Actual (m)}}$

The actual office was 15 m long.

Calculate the acutal width.

$$\frac{0.3}{w} = \frac{1}{15}$$

$$4.5 = w$$

$\frac{\text{Drawing (cm)}}{\text{Actual (m)}}$

The actual office was 4.5 m wide.

2. Scale: 1 cm represents 250 km.

a. The distance from Charlottetown to Halifax on the map is 0.7 cm.

$$\frac{0.7}{d} = \frac{1}{250}$$

$$175 = d$$

$\frac{\text{Drawing (cm)}}{\text{Actual (km)}}$

The actual distance is 175 km.

b. The distance from Quebec City to St. John's on the map is 6 cm.

$$\frac{6}{d} = \frac{1}{250}$$

$$1500 = d$$

$\frac{\text{Drawing (cm)}}{\text{Actual (km)}}$

The actual distance is 1500 km.

Section 11: Practice Activity 2

1. 130% of $6 = 1.3 \times 6$
$= 7.8$

130% of $4 = 1.3 \times 4$
$= 5.2$

The photocopied drawing will be 7.8 cm by 5.2 cm.

2. a. $\frac{4}{8} = \frac{1}{2}$
$= 0.5$
$= 50\%$

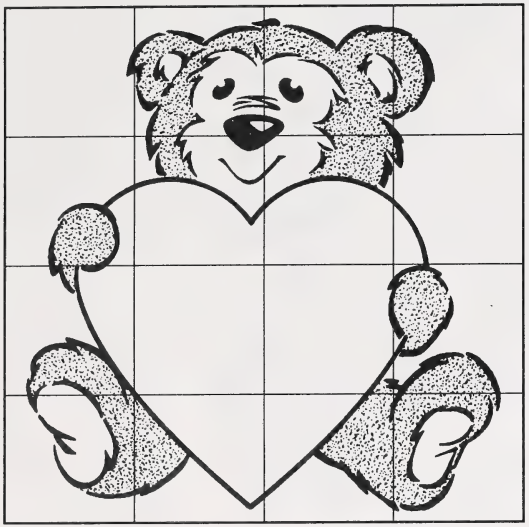
Helmut should select the 50% setting.

b. $\frac{10}{8} = \frac{5}{4}$
$= 1.25$
$= 125\%$

Helmut should select the 125% setting.

Section 11: Practice Activity 3

Step 1: Place a piece of tracing paper over the drawing and then draw a grid on the tracing paper.



Step 2: The original drawing is 8 cm by 8 cm. Calculate the size of the enlargement.

$$200\% \text{ of } 8 = 2 \times 8$$
$$= 16$$

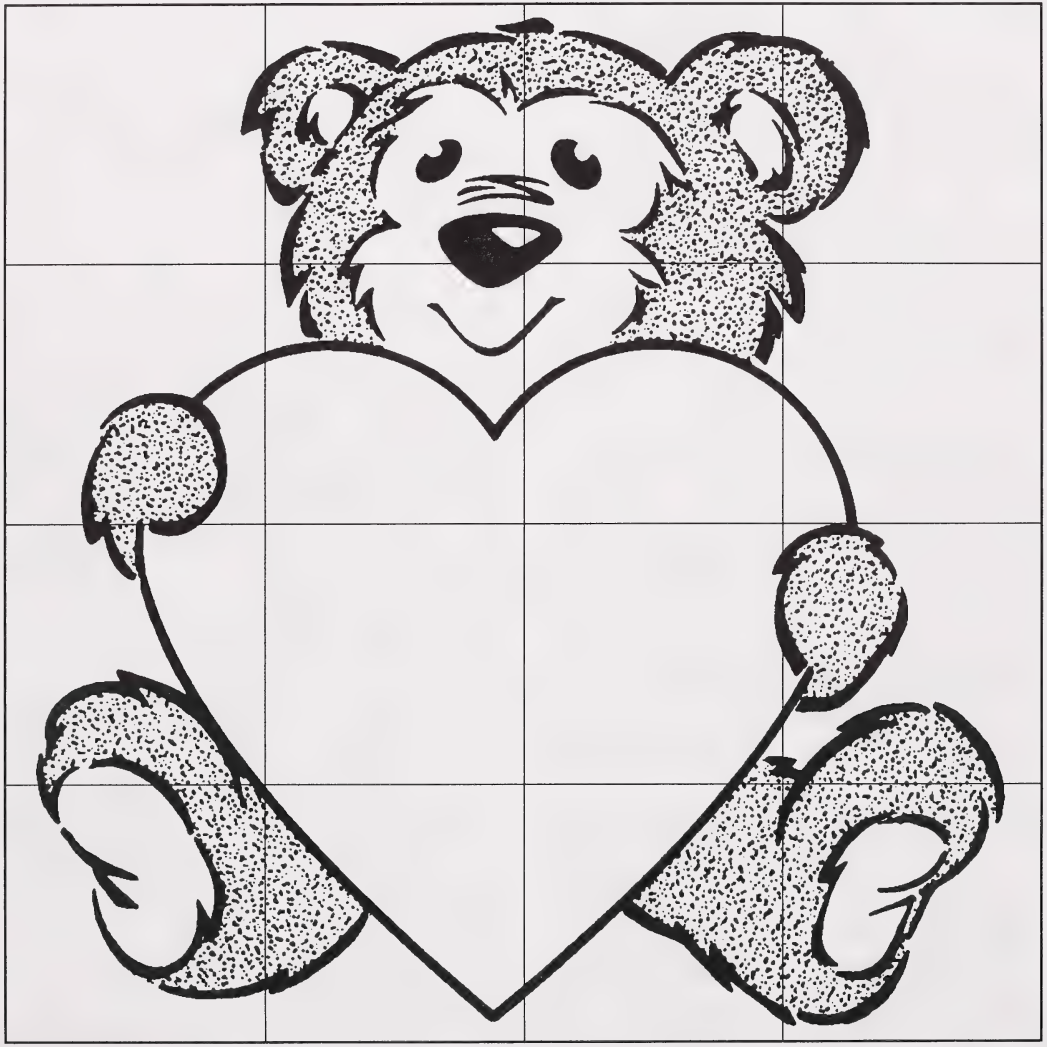
The enlargment will be 16 cm by 16 cm.

Step 3: Draw a box that is 16 cm by 16 cm and lightly draw the lines of a grid that corresponds to the original.

Step 4: Copy the picture from each square on the original grid to the corresponding square on the enlarged grid.

Step 5: Erase the grid lines.

Note: The enlargement is shown on the next page.



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Mathematics 9

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Module 5

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